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ABSTRACT

This guide was prepared as an instructional aid for teachers of a first-year course in Algebra. It was designed to be applicable to the wide range of Algebra 1 programs being offered in the junior and senior high schools. In this guide, the content of the Algebra 1 program has been divided into 11 major units, each unit organized around student performance objectives. It is intended that the performance objectives serve as guidelines for teachers to design the specific instructional program to meet the unique developmental needs of their students. Each of the 11 major units and four enrichment units include the following features: (1) a list of performance objectives; (2) a vocabulary list; (3) a list of key skills; (4) a list of textbook references; (5) four assessment tasks for each performance objective; (6) an answer key for the assessment tasks; (7) a list of entering performance objectives covering skills which should be reviewed before beginning the unit; (8) a list of assessment items for entering performance objectives; (9) an answer key; (10) a list of suggestions to the teacher, including the number of instructional days per unit; and (11) a list of suggested minimal, average, and maximal objectives for adjusting the course to different ability levels. (Author/MK)

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ALGEBRA 1

INSTRUCTIONAL GUIDE

Winter 1980

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Rockville, Maryland

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POINT OF VIEW FOR MATHEMATICS EDUCATION IN MONTGOMERY COUNTY

A provocative activity which teachers often use with pupils at various levels is that of trying to imagine what a world without numbers would be like. Such a world is difficult to imagine. The idea of number continues to play an important role in virtually all aspects of our world; and mathematics, therefore, constitutes a program of considerable importance in the schools.

As a discipline, mathematics is truly the art and science of abstraction. Characteristics of the physical world are converted into abstract ideas and symbols; these are then manipulated through mathematical operations to produce information and theorems about less easily observed aspects of the world. Recent evidence supports the contention that children's experiences with concrete materials are vital to later conceptual development. The school program thus proceeds from the concrete to the abstract.

The concepts of mathematics acquire greater meaning when they can be applied to the world in which we live. Because the variety and extent of mathematical applications have grown so rapidly in recent years, it is impossible for any one person to be conversant with the entire field. The school program must therefore be developed so that mathematical applications are selected and presented as efficiently as possible and with the intent of challenging pupils at all levels to see mathematics as an independent discipline as well as a tool for the advancement of other disciplines.

The Montgomery County mathematics program is designed and implemented to take into account the logical and relatively sequential nature of mathematics. Equally important, too, is the realization that the rate at which individual students learn mathematics varies significantly. The mathematics program is structured to encourage various approaches which allow students to progress at their individual rates.

The pre-algebra objectives range over six areas of mathematics and are arranged according to 14 different levels of achievement. Assessment measures have been constructed for each objective so that individual progress can be measured in a variety of categories. Enrichment activities are available for both the able student and the student who needs reinforcement.

Several options are available to the student who has completed the pre-algebra objectives. Differentiated paths through a variety of courses are available to the student, as can be seen in the Mathematics Program Patterns Chart, on page xv Each student has available a sequence of courses which can be suited to his/her interests and abilities.

Computer literacy is addressed at several levels of the mathematics program; career information is incorporated as appropriate throughout. Consumer applications are taught as mathematical skills are developed; the mathematics of consumerism is further emphasized in an elective senior high course.

In general terms, the instructional program in mathematics should help each student to:

Develop basic skills in using the vocabulary and symbols of mathematics

Develop skills in recognizing common geometric shapes

Develop basic skills in computing

Develop basic skills in working with geometric shapes

Develop basic skills in measuring, graphing, and using tables and charts

Develop understanding of the vocabulary and symbols of mathematics

Develop understandings necessary for translating among mathematical symbols, words, and the physical world

Develop concepts related to common geometric shapes

Develop understanding of computation

Develop understanding of measurement

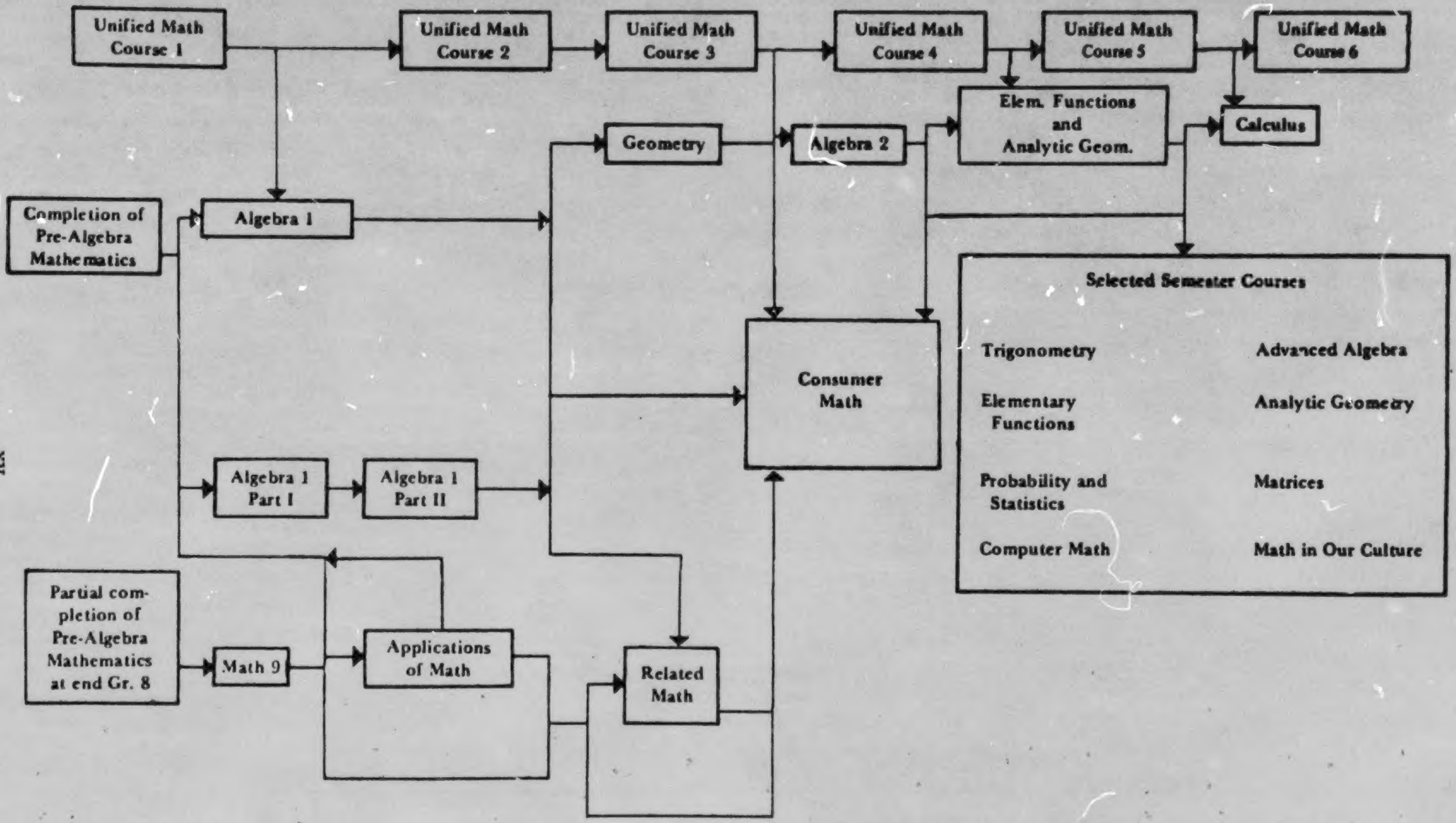
Develop an understanding of basic principles related to the structure of mathematics

Develop understanding and basic skills in problem solving

Apply the principles of mathematical reasoning to the solution of problems

Appreciate the significance of mathematics in daily living and its contribution to our cultural heritage

Use mathematics as needed in daily living


Unified Math Course 1
Unified Math Course 2
Unified Math Course 3
Unified Math Course 4
Unified Math Course 5
Unified Math Course 6
Elem. Functions and Analytic Geom.
Calculus
Geometry
Algebra 2
Completion of Pre-Algebra Mathematics
Algebra 1
Selected Semester Courses
Trigonometry
Advanced Algebra
Elementary Functions
Analytic Geometry
Probability and Statistics
Matrices
Computer Math
Math in Our Culture
Consumer Math
Algebra 1 Part I
Algebra 1 Part II
Partial completion of Pre-Algebra Mathematics at end Gr. 8
Math 9
Applications of Math
Related Math

MATHEMATICS PROGRAM PATTERNS CHART

OVERVIEW

PHILOSOPHY

The beginning algebra student is seen on the initial rung of the ladder leading to an understanding of the developmental structure of a mathematical system. Throughout the course, the development of an ability to think mathematically is emphasized as the student is guided toward an appreciation of the orderliness of mathematics. This appreciation greatly facilitates the acquisition of skills and techniques used to analyze and solve both simple and complex problems.

The study of Algebra 1 affords the student the opportunity to organize his/her own knowledge of mathematics and to expand skills previously learned. Mastery of algebra is seen as the passport for a student's journey through the complexities of higher mathematics.

INTENT

This guide has been prepared as an instructional aid for teachers of a first year course in algebra. It has been designed to be applicable to the wide range of Algebra 1 programs being offered in the junior and senior high schools. In this guide, the content of the Algebra 1 program has been divided into eleven major units, each unit organized around student performance objectives. It is intended that the performance objectives serve as guidelines for teachers to design the specific instructional program to meet the unique developmental needs of their students.

ORGANIZATION AND FEATURES

Each of the eleven major units and four enrichment units deals with the development of the skills and concepts of a topic of first year algebra. With each unit, the following features are included:

- A list of performance objectives for the unit, each objective categorized by level of Bloom's taxonomy (See page xix.)
- A vocabulary list for the unit
- A list of key skills for end-of-course testing (See page xxii.)
- A list of textbook references for each performance objective
- Four assessment tasks for each performance objective, which can be used for testing or review
- An answer key for the assessment tasks
- A list of entering performance objectives covering skills which should be reviewed before beginning the unit (Note: These objectives include pre-algebra skills as well as skills from prior units.)
- A list of assessment items for each entering performance objective

- An answer key for the entering performance objectives assessments

- A list of suggestions to the teacher including the number of instructional days per unit. (Note: (1) The suggested number of instructional days does not include provisions for review or testing. (2) Memory aids included in the suggestions are intended to facilitate retention, not to take the place of learning the mathematical theory.)

- A list of suggested minimal, average, and maximal objectives for adjusting the course to different levels of student ability

SEQUENCE

In examining all Algebra 1 books currently approved for use in the county, it was observed that they each developed the material in a slightly different order. After much discussion, the units of this guide were placed in order so that the study of linear relations and the language of algebra (Units I-VI) would constitute the first semester of the course, and the development of higher order relations and their applications would form the central theme of the second semester.

Many teachers prefer to introduce polynomials, factoring, and rational algebraic expressions (Units VII, VIII and IX) before graphing and systems of equations (Units V and VI). This guide reverses that order. It is left to the individual teacher to decide which sequence is best adapted to his/her particular needs.

KEY SKILLS FOR END-OF-COURSE TESTING

A list of twenty-eight summary skills, the testing of which would be a measure of the mastery of the Algebra 1 objectives as described in the MCPS Program of Studies, can be found on pages xxii and xxiii. The first fourteen skills cover the objectives of Algebra 1A; the last fourteen cover the objectives ot Algebra 1B.

In each unit of the Algebra 1 Instructional Guide, the key skills pertinent to that unit are listed on the page listing the performance the performance objectives for the unit.

LEVELS OF PERFORMANCE OBJECTIVES

Each performance objective in each unit has been classified as measuring knowledge (I), comprehension (II), or application (III) according to Bloom's taxonomy. While these classifications are highly subjective and open to interpretation, they have been included as an indication of the nature of the responses necessary to demonstrate attainment of the objectives. The following list of verbs was used in classifying each objective.

I. Knowledge

Emphasis: Recall

choose from a list (judgment not involved)
define (give a dictionary definition)
fill in the blank (or complete)
follow directions
identify
indicate
label
list
locate (on a map or a given document)
match
name
select (judgment not involved)

II. Comprehension

Emphasis: Grasp of meaning, intent, or relationship

classify
define (in student's own words)
describe
explain
express in other terms
find (as in math)
measure
paraphrase
put in order
recognize
rewrite
simplify
suggest
summarize
Math
add (find the sum)
balance
calculate
compute (using a given formula)
divide (find the quotient)
factor
find square root or raise to power
multiply (find the product)
perform operations on numbers
subtract (find the difference)

III. Application

Emphasis: Applying appropriate principles or generalizations

apply
collect information (supply correct equation or formula)
compute
construct
convert (in math)
draw
determine (calculate)
demonstrate
derive
differentiate between
discuss
distinguish between
expand
express in a discussion
estimate
find (implies investigation)
interpret
investigate
illustrate (give examples not previously specified)
graph
keep records
locate (information)
make
participate
plan
predict (from known factors)
prepare
present
prove (in math)
solve (problems expressed in words)
use
trace (development, history, process)
translate

Algebra 1 - Grades 9, 10, 11, 12

Prerequisite: Attainment of the pre-algebra objectives in Elementary and Pre-Algebra Mathematics Objectives, Bulletin 285

3100   1 year   1 credit

Emphasis in Algebra 1 is on the development of an understanding of the basic structure of algebra related to the real number system; a recognition of the techniques of algebra as structures of this system; facility in applying algebraic concepts and skills; perception of the role of deductive reasoning in algebra; and an appreciation of the need for precision of language.

As techniques are developed, applications to appropriate, relevant problems are made; e.g., problems involving practical geometry, number theory, weather, air navigation, and money are included. Opportunities for enriching work in greater depth occur throughout the program.

Algebra 1 is taught as a one-year course or as a two-year course as student needs indicate; pupil placement depends on a decision made cooperatively by the student, the parent, and the school staff.

Algebra 1-A

Prerequisite: Attainment of the pre-algebra objectives in Elementary and Pre-Algebra Mathematics Objectives, Bulletin 285

3111   1 semester   ½ credit

Upon completion of Algebra 1-A, the student should be able to:

- demonstrate the use of the roster, rule, and graphing methods in representing sets
- compute the value of a numerical expression involving symbols of inclusion and the order of operations rules
- add, subtract, multiply, and divide directed numbers
- apply properties of equality and inequality and the concepts of additive and multiplicative inverse to solve equations and inequalities
- apply the fundamental operations to solve open sentences and word problems involving real numbers
- graph the solution sets of linear equations and inequalities

- add, subtract, multiply, and divide polynomials
- factor polynomials completely
- solve equations involving polynomials
- solve word problems involving the factoring of polynomials

Algebra 1-B

Prerequisite: Attainment of the objectives of Algebra 1-A

3112	1 semester	½ credit

Upon completion of Algebra 1-B, the student should be able to:

- rewrite a linear equation into slope-intercept forms and graph its solution set
- determine an equation of a line
- solve a system of open sentences into variables
- solve word problems involving open sentences in two variables
- add, subtract, multiply, and divide algebraic fractions
- write radical expressions in simplest form
- solve quadratic equations
- solve equations and word problems involving irrational numbers

Algebra 1, Parts I and II

3101 (Part I)	1 year	1 credit per year (9-12)
3102 (Part II)	1 year	

Algebra 1, Parts I and II, is a two-year program for achieving the Algebra 1 objectives. This program is offered for students who have satisfactorily completed necessary pre-algebra objectives but who need additional instruction and time to achieve understanding of abstract concepts. Textbooks written especially for a two-year program are provided. The objectives for Algebra 1, Part I, are those for Algebra 1-A. The objectives for Algebra 1, Part II, are those for Algebra 1-B.

KEY SKILLS FOR END-OF-COURSE TESTING

As identified in the Algebra 1 Instructional Guide and the credit for examination, the student should be able to:

1. Demonstrate the use of the roster, rule, and graphing methods in representing sets
2. Compute the value of numerical expression involving symbols of inclusion and the order of operations rules
3. Evaluate algebraic expressions and open sentences by substituting for the variable
4. Add and subtract directed numbers
5. Multiply and divide directed numbers
6. Combine similar terms
7. Solve linear equations in one variable by applying the properties of equality
8. Solve linear inequalities in one variable by applying the properties of inequality
9. Solve various types of word problems, utilizing an organized approach
10. Graph linear equations in the coordinate plane
11. Graph linear inequalities in the coordinate plane
12. Determine an equation of a line, given the slope and a point or two points of the line
13. Solve a system of equations in two variables
14. Solve word problems involving two variables and a system of equations
15. Add and subtract polynominials
16. Multiply polymonials
17. Divide polynomials
18. Solve linear equations involving polynomials
19. Factor polynomials completely
20. Solve equations by factoring

21. Solve word problems involving factoring

22. Simplify an algebraic fraction

23. Multiply and divide algebraic fractions

24. Add and subtract algebraic fractions

25. Solve fractional equations

26. Write radical expressions in simplest form

27. Solve radical equations

28. Solve a quadratic equation by completing the square or applying the quadratic formula

BASIC TEXTBOOKS FOR ALGEBRA 1 AND THE TWO-YEAR ALGEBRA COURSE

1. Dolciani, Mary P.; Wootton, William; and Beckenbach, Edwin. Algebra I. Boston: Houghton Mifflin Company, 1978.

2. Dolciani, Mary P., and Wootton, William. Algebra Structure and Method Book 1. Boston: Houghton Mifflin Company, 1976.

3. Foster, Alan G.; Roth, James N.; Winters, Leslie J. Algebra One. Columbus, Ohio: Charles B. Merrill Publishing Company, 1979.

4. Jacobs, Russell. Discovering Algebra I. New York: Harcourt Brace Jovanovich, Inc., 1974.

5. Keedy, Mervin; Bittinger, Marvin; and Smith, Stanley. Algebra One. Menlo Park, Ca: Addison-Wesley Publishing Company, 1978.

6. Payne, Joseph; Coxford, Arthur; Lankford, Francis, Jr.; and Zamboni, Floyd. Algebra I. New York: Harcourt Brace Jovanovich, Inc., 1977.

7. Sobel, Max A., and Banks, J. Houston. Algebra Book 1. New York: Webster Division, McGraw Hill Book Company, 1977.

8. Travers, Kenneth J.; Dalton, Leroy C.; and Brunner, Vincent. Using Algebra. River Forest, Illinois: Laidlaw Brothers, 1974.

ALGEBRA 1, PART I AND PART II. SETS

1. Denholm, Richard; Underhill, Robert; and Dolciani, Mary P. Elementary Algebra 1, 2. Boston: Houghton Mifflin Company, 1977.

2. Jacobs, Russell F. Introductory Algebra 1, 2. New York: Harcourt Brace Jovanovich, Inc., 1976.

3. Johnson, Richard E.; Johnson, Cheryl G.; and Bakhru, Fay. Algebra I-A, Two-Part Course, Part 1, 2. Menlo Park, Ca: Addison Wesley Publishing Company, 1977.

OUTLINE OF COURSE CONTENT

BASIC UNITS

I. SETS

II. FUNDAMENTAL CONCEPTS

III. REAL NUMBERS: PROPERTIES AND OPERATIONS

IV. SOLVING OPEN SENTENCES AND WORD PROBLEMS

V. GRAPHING

VI. SYSTEM OF OPEN SENTENCES

VII. POLYNOMIALS

VIII. FACTORING

IX. RATIONAL ALGEBRAIC EXPRESSIONS

X. RADICAL EXPRESSIONS

XI. QUADRATIC EQUATIONS

SUGGESTED ENRICHMENT UNITS

XII. NUMERICAL TRIGONOMETRY

XIII. PERIMETERS, AREAS, AND VOLUMES

XIV. RATIO, PROPORTION, AND PERCENT

XV. FLOWCHARTS

UNIT I - SETS

PURPOSE

This unit is designed to provide the students with a background in the use of sets. The emphasis is on the application of terminology and set notation as a building block for the study of elementary algebra. While some texts include sets in a unit on fundamental concepts, it has been separated in this guide to provide a short, initial unit for the opening days of school.

OVERVIEW

Familiarization with set notation and the methods of specifying sets are encompassed in the first several objectives. Recognition of various types of sets and set relationships is expected, as is a knowledge of the operations on sets.

SUGGESTIONS TO THE TEACHER

Instructional Days: 4-5
Minimal Course Objectives #1-5, 10, 11
Average Course Objectives #1-11
Maximal Course Objectives: ALL

Writing accurate descriptions of sets given in roster form can be difficult for some students. The students should be given a wide range of non-numerical as well as numerical sets to describe.

Emphasis should be given to the concepts of union and intersection of sets since these ideas are necessary for future units.

For many students, this unit is a review and should be treated as such.

VOCABULARY

between
complement
coordinate
counting numbers
element
empty set
equal sets
factor
finite set
graph
inclusive
infinite set
intersection
integer
member
natural numbers
null set
number line
one-to-one correspondence
prime factor
prime numbers
real numbers
roster
rule
set
subset
union
universal set
whole numbers

PRE-ENTRY DIAGNOSTIC ARITHMETIC TEST

Add whole numbers.

1. $\begin{array}{r}97\\ \underline{68}\end{array}$ 2. $\begin{array}{r}507\\ \underline{946}\end{array}$ 3. $\begin{array}{r}4284\\ \underline{6998}\end{array}$

Subtract whole numbers.

4. $\begin{array}{r}759\\ \underline{508}\end{array}$ 5. $\begin{array}{r}725\\ \underline{284}\end{array}$ 6. $\begin{array}{r}50{,}901\\ \underline{26{,}978}\end{array}$

Multiply whole numbers.

7. $\begin{array}{r}80\\ \underline{32}\end{array}$ 8. $\begin{array}{r}473\\ \underline{65}\end{array}$ 9. $\begin{array}{r}843\\ \underline{726}\end{array}$

Divide whole numbers.

10. $38\overline{)\,419}$ 11. $46\overline{)\,3198}$ 12. $29\overline{)\,1889}$

Add decimals.

13. $\begin{array}{r}11.84\\ \underline{+\ 4.95}\end{array}$ 14. $7.305 + 8.45$

15. $9.6 + 12.07 + 15 + 23.297$

Subtract decimals.

16. $\begin{array}{r}83.24\\ \underline{15.77}\end{array}$ 17. $26 - 3.2$ 18. $38.29 - 0.7$

Multiply decimals.

19. 8.8×7.7 20. $4830 \times .073$ 21. 0.924×34.5

Divide decimals.

22. $8\overline{)\,62.4}$ 23. $6.9\overline{)\,3.45}$ 24. $.068\overline{)\,333.2}$

Reduce fractions to lowest terms.

25. $\frac{24}{27}$ 26. $\frac{18}{54}$ 27. $\frac{45}{60}$

Change mixed numbers to improper fractions.

28. $7\frac{5}{6}$ 29. $3\frac{7}{9}$ 30. $5\frac{3}{4}$

Change improper fractions to mixed numbers.

31. $\frac{27}{5}$ 32. $\frac{31}{12}$ 33. $\frac{8}{3}$

Multiply fractions.

34. $\frac{7}{11} \cdot \frac{5}{9}$ 35. $\frac{7}{8} \cdot 3\frac{4}{7}$ 36. $5\frac{5}{6} \times 2\frac{9}{10}$

Divide fractions.

37. $\frac{4}{7} \div \frac{3}{5}$ 38. $5\frac{5}{9} \div \frac{2}{7}$ 39. $5\frac{5}{8} \div 5\frac{5}{6}$

Add fractions.

40. $\frac{2}{5} + \frac{3}{4}$ 41. $3\frac{4}{5} + 7\frac{3}{10}$ 42. $2\frac{5}{6} + 4\frac{2}{3}$

Subtract fractions.

43. $\frac{11}{15} - \frac{1}{2}$ 44. $5\frac{3}{8} - 2\frac{1}{4}$ 45. $3\frac{1}{8} - 2\frac{5}{6}$

PRE-ENTRY DIAGNOSTIC ARITHMETIC TEST ANSWER KEY

1. 165
2. 1453
3. 11,282
4. 251
5. 441
6. 23,923
7. 2,560
8. 30,745
9. 612,018
10. $11\frac{1}{38}$
11. $69\frac{12}{23}$
12. $65\frac{4}{29}$
13. 16.79
14. 15.755
15. 59.967
16. 67.47
17. 22.8
18. 37.59
19. 67.76
20. 352.59
21. 31.878
22. 7.8
23. .5
24. 4,900
25. $\frac{8}{9}$
26. $\frac{1}{3}$
27. $\frac{3}{4}$
28. $\frac{47}{5}$
29. $\frac{34}{9}$
30. $\frac{23}{4}$
31. $5\frac{2}{5}$
32. $2\frac{7}{12}$
33. $2\frac{2}{3}$
34. $\frac{35}{99}$
35. $\frac{25}{8}$ or $3\frac{1}{8}$
36. $16\frac{11}{12}$
37. $\frac{20}{21}$
38. $\frac{175}{9}$ or $19\frac{4}{9}$
39. $\frac{27}{28}$
40. $\frac{23}{20}$ or $1\frac{3}{20}$
41. $11\frac{1}{10}$
42. $7\frac{1}{2}$
43. $\frac{7}{30}$
44. $3\frac{1}{8}$
45. $\frac{7}{24}$

UNIT I - SETS

PERFORMANCE OBJECTIVES

1. Identify the following symbols: $=$, $\in$, $\notin$, $\emptyset$, $\{\ \}$, $\cap$, $\cup$, $\subset$. (I)
2. State whether a given number is an element of a given set. (II)
3. Write a roster for a set when given its rule. (II)
4. Determine a rule for a set when given its roster. (III)
5. Graph a given set on a horizontal number line. (III)
6. List all subsets of a given set. (II)
7. Distinguish between finite and infinite sets. (III)
8. Determine whether a one-to-one correspondence exists between two given sets. (III)
9. Determine whether two given sets are equal. (III)
10. Find the intersection of two given sets. (III)
11. Find the union of two given sets. (III)
12. Construct Venn diagrams to show the relationship between sets. (III)
13. Find the complement of a given set. (III)

Minimal	Average	Maximal
#1-5, 10, 11	#1 - 11	All

KEY SKILLS FOR END-OF-COURSE TESTING

1. Demonstrate the use of the roster, rule and graphing methods in representing sets.

UNIT I - SETS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	X	21-26 168-171	4	1-5 39-42 131-138	471-474 536	11,13 107,108	1-8	1-5 130-133
2	17	26	4	---	471	---	1-8	2
3	---	24-26	4	1-5	474	---	1-8	4
4	---	24-26	4	3-5	474	---	1-8	4
5	31-32 258-259	27-30	58	---	54-56	---	18 109-111	9-11 15,18
6	258-259	25-26	4	2,4,6	473-474	---	7-8	5-7
7	258-259	27-30	--	3-5	470	446,450	1-4	2-3
8	33	---	--	---	490	---	17	6-7
9	258	25	--	---	---	---	---	6-7
10	270-272	168-171	42	131-138	480-482	107-108	113	130-133
11	270-272	168-171	42	131-138	480-482	107-108	115-116	130-133
12	270-272	168-171	42	---	480-482	450	157-160	130
13	---	---	--	---	---	---	158	---

UNIT I - SETS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)
1	XII	86,98 101	---	1-10
2	---	87	108	1
3	---	86-87	108	7
4	---	86-87	108	279
5	37-47	86-87	---	2,127
6	---	207-208	---	24,127
7	---	385	---	---
8	---	---	106	---
9	---	---	---	---
10	---	101	384-388	---
11	---	98	384-388	---
12	---	---	---	---
13	---	---	---	---

PERFORMANCE OBJECTIVE I-1

Identify the following symbols: $=$, $\neq$, ϵ, $\not\epsilon$, $\emptyset$, $\{\ \}$, $\cup$, $\cap$, $\subset$.

a) Which of the following statements is incorrect?

A. $\{1, 2\} \subset \{1, 2\}$

B. $2 \epsilon \{1, 2, 3\}$

C. $\{3\} \epsilon \{1, 2, 3\}$

D. $\{a, b, c\} \neq \{1, 2, 3\}$

Answer_______________

b) Place one of the symbols $=$, $\neq$, ϵ, $\not\epsilon$, $\cup$, $\cap$, $\subset$, in the blank to make each statement true.

(1) 3 _ $\{1, 2, 3\}$

(2) $\{3\}$ _ $\{1, 2, 3\}$

(3) $\{1, 2, 3\}$ _ $\{3, 1, 2\}$

(4) $\{a, b, c\}$ _ $\{c, d, e\} = \{c\}$

c) Match each of the following symbols with its description.

_____(1) $\cup$ A. is a subset of

_____(2) $\subset$ B. is an element of

_____(3) ϵ C. union

_____(4) $\cap$ D. intersection

d) Match each of the following symbols with its description.

_____(1) $\subset$ A. the empty set

_____(2) $\emptyset$ B. is not equal to

_____(3) $\neq$ C. is a subset of

_____(4) ϵ D. is an element of

PERFORMANCE OBJECTIVE I-2

State whether a given number is an element of a given set.

a) Place either $\in$ or $\notin$ in the blank to make each statement true.

(1) 3 _____ $\{1, 2, 3\}$

(2) 1 _____ {prime numbers}

(3) 0 _____ {natural numbers}

b) Which of the following is false?

A. 3 $\in$ {odd numbers}

B. 4 $\in$ {prime numbers}

C. 0 $\notin$ {counting numbers}

D. Rockville $\notin$ {counties in the state of Maryland}

Answer _______________

c) Given the following:

(1) 0 $\in$ {whole numbers}

(2) 0 $\notin$ {natural number}

which of the following statements is correct?

A. only #1 is true

B. only #2 is true

C. both #1 and #2 are true

D. both #1 and #2 are false

Answer _______________

d) True or false

_____ (1) 1 $\in$ {prime numbers}

_____ (2) 0 $\notin$ {natural numbers}

_____ (3) $3\frac{1}{2} \in$ {whole numbers}

_____ (4) 8 $\in$ {whole numbers between 3 and 8}

PERFORMANCE OBJECTIVE I-3

Write a roster for a set when given its rule.

a) Write the roster for {states bordering Washington, D.C.}.

Answer_______________

b) Write the roster for {grade levels in this school}.

Answer_______________

c) Write the roster for {whole numbers between 1 and 2}.

Answer_______________

d) Which of the following is the correct roster for {positive integral multiples of 5} ?

A. $\{5, 10, 15, 20, \ldots\}$

B. $\{5, 10, 15, 20\}$

C. $\{0, 5, 10, 15, 20, \ldots\}$

D. $\left\{\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \ldots\right\}$

Answer_______________

PERFORMANCE OBJECTIVE I-4

Determine a rule for a set when given its roster.

a) State a rule that describes the following set: $\{2, 4, 6, 8\}$.

Answer ______________

b) State a rule that describes the following set: {January, June, July}.

Answer ______________

c) State a rule that describes the following set $\{2, 3, 5, 7\}$.

Answer ______________

d) Which of the following best describes the set $\{1, 3, 5, 7, \ldots\}$?

A. {odd numbers less than 8}

B. {odd numbers greater than zero}

C. {odd numbers between zero and 8}

D. None of the above

Answer ______________

PERFORMANCE OBJECTIVE I-5

Graph a given set on a horizontal number line.

a) Graph {1, 3, 5} on the number line.

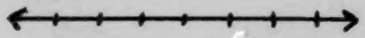
Answer

b) Graph {whole numbers between 1 and 3} on the number line.

Answer

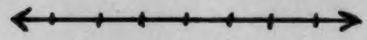
c) Graph {real numbers between 0 and 4} on the number line.

Answer



d) Graph {counting numbers between -3 and 5} on the number line.

Answer



PERFORMANCE OBJECTIVE I-6

List all subsets of a given set,

a) List all the subsets of {1, 2, 3}.

Answer______________

b) The set {a, b, c} has __?__ subsets.

A. 3

B. 6

C. 7

D. 8

Answer______________

c) Which of the following sets is not a subset of {1, 2, 3} ?

A. {3, 1}

B. {1, 2, 3}

C. {∅}

D. {1, 2}

Answer______________

d) Which is not a subset of {whole numbers}?

A. {natural numbers}

B. {2, 4, 6, 8, ...}

C. {1, 1, 2, 3, 5, 8, 13, ...}

D. {real numbers}

Answer ______________

PERFORMANCE OBJECTIVE I-7

Distinguish between finite and infinite sets.

State whether each of the following is finite or infinite.

Answer

a) {students in this school} ______________

b) {real numbers between 2 and 4} ______________

c) {grains of sand on the beach at Ocean City, Maryland} ______________

d) {2, 4, 6, ... 20} ______________

e) {whole numbers between 3 and 4} ______________

f) {fractions between $\frac{1}{4}$ and $\frac{1}{2}$} ______________

PERFORMANCE OBJECTIVE I-8

Determine whether a one-to-one correspondence exists between two given sets.

a) Write a set that could be in a one-to-one correspondence to {even numbers between 0 and 10}.

Answer______________

b) Which of the following sets could be in a one-to-one correspondence with {whole number multiples of 3 between 0 and 10}?

A. {a, b, c}

B. {3}

C. {1, 2, 3, 4, 5, 6, 7, 8, 9}

D. {a, b, c, d}

Answer______________

c) Which of the following sets could be in a one-to-one correspondence with {○, Δ, □}.

A. {1, 2, 3}

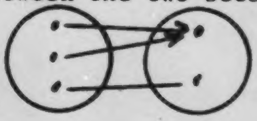
B. {○, Δ}

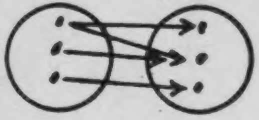
C. {3}

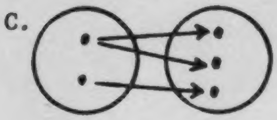
D. None of the above

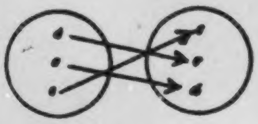
Answer______________

d) Which of the following diagrams shows a one-to-one correspondence between the two sets?

A.


B.


C.


D.


Answer______________

PERFORMANCE OBJECTIVE I-9

Determine whether two sets are equal.

a) Which of the following sets is equal to { letters in the word that } ?

A. { 2, 3, 5, 7 }

B. { letters in the word hat }

C. { a, b, c, d }

D. { a, b, c }

Answer______________

b) Write a set which is equal to { letters in the word Mississippi }.

Answer______________

c) Which of the following sets is equal to {prime factors of 12} ?

A. {1, 2, 3, 4, 6, 12}

B. {2, 3 }

C. {1, 2, 3}

D. {2, 3, 6 }

Answer______________

d) Which of the following sets is equal to the set { A, C, E } ?

A. {1, 2, 3}

B. {C, E, A}

C. {○, △, □ }

D. All of the above

Answer______________

PERFORMANCE OBJECTIVE I-10

Find the intersection of two given sets.

a) Determine the intersection of $\{1, 2, 3, 4\}$ and $\{3, 4, 5, 6\}$.

Answer______________

b) Given: $A = \{b, c, d\}$

$B = \{c, d, e, f\}$,

then $A \cap B =$ ______________.

c) Given $A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

then $A \cap B =$ ______________.

d) Given $I = \{1, 2, 3, 4, 5\}$

$J = \{2, 4, 6\}$

$K = \{1, 3, 5\}$,

then $I \cap J =$ ______________

$J \cap K =$ ______________

and $(I \cap J) \cap K =$ ______________

PERFORMANCE OBJECTIVE I-11

Find the union of two given sets.

a) Given: A = {1, 2, 3, 4}

B = {3, 4, 5, 6}

then A ∪ B = _____.

A. {3, 4}

B. {1, 2, 3, 3, 5, 6}

C. {1, 2, 3, 4, 5, 6}

D. ∅

b) Given: A = {letters in the word hot}

B = {letters in the word shoot},

find A ∪ B.

Answer_______________

c) Given: A = {13, 17, 23, 29}

B = {prime numbers less than 25},

find A ∪ B.

Answer_______________

d) Given: A = {4, 8, 12, 16}

B = {8, 16, 24},

then A ∪ B = __________.

A. {8, 16}

B. {4, 8, 8, 12, 16, 16}

C. {4, 8, 12, 16, 24}

D. ∅

PERFORMANCE OBJECTIVE I-12

Construct Venn diagrams to show the relationship between sets.

a) Construct a Venn diagram to show $A \cap B$.

Answer

b) Construct a Venn diagram to show $A \cap B = \emptyset$.

Answer

c) Construct a Venn diagram to show $A \subset B$.

Answer

d) Construct a Venn diagram to show $A \cup 3$.

Answer

PERFORMANCE OBJECTIVE I-12 (continued)

Construct Venn diagrams to show the relationship between sets.

e) Construct a Venn diagram which shows the relationship between the following sets:

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{4, 5, 6, 7, 8, 9\}$

$C = \{3, 4, 5, 7, 10\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Answer

PERFORMANCE OBJECTIVE I-13

Find the complement of a given set .

a) Given that $A \subset$ {whole numbers} and A = {even whole numbers}, find the complement of A.

Answer______________

b) Given: $A \subset$ {whole numbers}

A = {positive and negative whole numbers},

find the complement of A.

Answer______________

c) Given that $A \subset$ {real numbers} and A = {positive real numbers} , find $\overline{A}$.

Answer______________

d) Given: U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} ,

A = {2, 4, 6, 8, 10} ,

B = {1, 3, 5, 7, 9} ,

C = {2, 3, 4}, and

D = {1, 2, 3, 7, 8, 9} ,

find:

(1) $\overline{A}$ =

(2) $\overline{B}$ =

(3) $\overline{C}$ =

(4) $\overline{D}$ =

UNIT I - SETS

Answers

1. a) C

   b) 1. $\in$

   2. $\subset$

   3. $=$

   4. $\cap$

   c) 1. C

   2. A

   3. B

   4. D

   d) 1. C

   2. A

   3. B

   4. D

2. a) 1. $\in$

   2. $\notin$

   3. $\notin$

   b) B

   c) C

   d) 1. F

   2. T

   3. F

   4. F

3. a) {Maryland, Virginia}

   b) answer varies

   c) $\emptyset$

   d) A

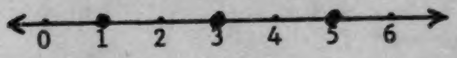
4. a) {even numbers between 1 and 2}

   b) {months of the year beginning with J}

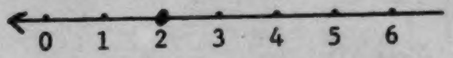
   c) {prime numbers less than 10}

   d) B

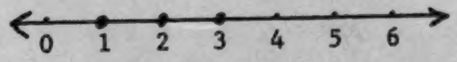
5. a)



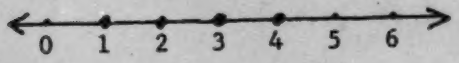
b)



c)



d)



6. a) {1} {1, 2} {1, 2, 3}

   {2} {1, 3} $\emptyset$

   {3} {2, 3}

   b) D

   c) C

   d) D

7. a) finite

   b) infinite

   c) finite

   d) finite

   e) finite

   f) infinite

Answers (continued)

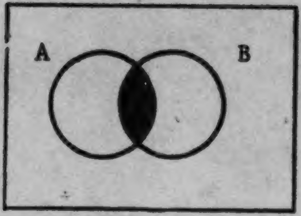
8. a) Any set of 4 elements
   b) A
   c) A, B
   d) D

9. a) B
   b) {p, i, s, m}
   c) B
   d) B

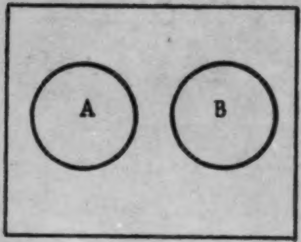
10. a) {3, 4}
    b) {c, d}
    c) ∅
    d) 1. {2, 4}
       2. ∅
       3. ∅

11. a) C
    b) {s, h, o, t}
    c) {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
    d) C

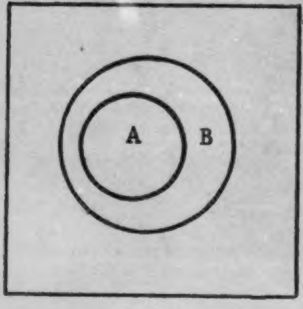
12. a)



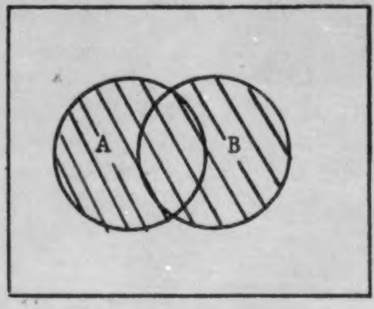
b)



c)



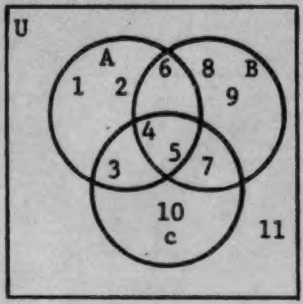
d)



Answers (continued)

12. (continued)

e)


U
A
B
c
1
2
6
8
9
4
5
3
7
10
11

13. a) {odd whole numbers}

b) {0}

c) {0, negative real numbers}

d) 1. {1, 3, 5, 7, 9}

2. {2, 4, 6, 8, 10}

3. {1, 5, 6, 7, 8, 9, 10}

4. {4, 5, 6, 10}

UNIT II - FUNDAMENTAL CONCEPTS

PURPOSE

This unit serves as a foundation for the development of elementary algebra. Through mastery of the symbols, terminology, and computational conventions, the students are prepared to use the language of algebra.

OVERVIEW

Computations involving order of operations and symbols of inclusion rules are stressed. These skills are applied to the evaluation of algebraic expressions for specific replacements of the variable. These two concepts are then integrated into the translation of word phrases into algebraic expressions. In this unit and subsequent units, open sentence refers to both equations and inequalities. The word coefficient refers to the numerical coefficient.

SUGGESTIONS TO THE TEACHER

Instructional Days: 6-7
Minimal Course Objectives: ALL
Average Course Objectives: ALL
Maximal Course Objectives: ALL

The concepts of sets developed in unit one should be extended to include the ideas of domain and solution set.

The parallel between the strategies of learning algebra and learning a foreign language is helpful.

An analogy can be made between the use of punctuation marks in English and the use of symbols of inclusion in mathematics. Mnemonic devices and other memory aids can be used to help students retain some key ideas (example: My Dear Aunt Sally can help students remember that multiplications and divisions are performed before additions and subtractions in the order of operation).

VOCABULARY

algebraic expression
base
braces
brackets
coefficient
constant
cube
difference
domain
equation
evaluate
exponent
factor
inequality
number
numeral
numerical expression
open sentence
parentheses
power
product
quotient
replacement set
simplify
solution set (truth set)
square
substitution
sum
term
variable

UNIT II - FUNDAMENTAL CONCEPTS

ENTERING PERFORMANCE OBJECTIVES

1. Compute the value of an arithmetic expression using the order of operations.

2. Compute the value of an arithmetic expression containing symbols of inclusion.

3. Place the proper symbol, <, >, or =, between two numerical expressions to make a true statement.

4. Simplify arithmetic expressions containing exponents.

Assessment Tasks

1. Simplify each expression:

a) $27 - 5 \cdot 3 + 24 \div 3$ a) ______________

b) $18 \div \cdot \frac{1}{2} + 16 \cdot \frac{1}{2}$ b) ______________

c) $\frac{25 \times 4 + 20 \div 4}{3}$ c) ______________

d) $5 \times 8 \div 2 + 3 \times 6$ d) ______________

2. Simplify each expression:

a) $17 + [\,(8 + 14) \div 2] - 9$ a) ______________

b) $\frac{19 \cdot 3 - 10 \cdot 3}{4 + 5}$ b) ______________

c) $[\,(15 + 20) \div 5] + 6 \cdot 2^2$ c) ______________

d) $7\,\{(51 - 27) \div [2\,(9 + 3)\,]\}$ d) ______________

3. Insert <, >, or = to make a true statement:

a) $4 \cdot 0$ __________ $0 + 4$

b) $\frac{18 + 26}{2}$ __________ $\frac{5 \times 6 + 11 \times 13}{7 - 4}$

c) $10 + 3^2$ __________ $4^2 + 8$

d) $\frac{429}{3} + 7$ __________ $5 \times 5 \times 6$

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

4. Simplify each expression:

a) $\dfrac{2^2 + 3 - 12 \div 2}{3^2 - 2^3}$ a) ______________

b) $2^4 - 2^2 \div 2^2 - 2$ b) ______________

c) $2 \cdot 3^3 - 2 \cdot 3^2 - 3^2 + 5$ c) ______________

d) $6^3 \div 3^2 \div 2^2 + 7$ d) ______________

UNIT II - FUNDAMENTAL CONCEPTS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) 20
   b) 44
   c) 35
   d) 38

2. a) 19
   b) 3
   c) 31
   d) 7

3. a) <
   b) >
   c) <
   d) =

4. a) 1
   b) 13
   c) 32
   d) 13

UNIT II - FUNDAMENTAL CONCEPTS

PERFORMANCE OBJECTIVES

1. Identify the following:

    a) Factor
    b) Term
    c) Coefficient
    d) Exponent
    e) Base
    f) Power
    g) Variable
    h) Constant (I)

2. Compute the value of a numerical expression using the order of operations rules. (II)

3. Compute the value of a numerical expression punctuated with symbols of inclusion. (II)

4. Evaluate algebraic expressions when given numerical replacements for the variables. (II)

5. Place the proper symbol (= , > , <) between two numerical expressions to make a true statement. (II)

6. State the number of terms in a given algebraic expression. (II)

7. Translate word phrases into algebraic expressions. (III)

8. Determine the solution set of an open sentence by replacing the variable with elements of a given domain. (III)

Minimal	Average	Maximal
All	All	All

KEY SKILLS FOR END-OF-COURSE TESTING

2. Compute the value of a numerical expression involving symbols of inclusion and the order of operations rules.

3. Evaluate algebraic expressions and open sentences by substituting for the variable.

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	4 7-8	11,31 52,59	2, 13	14,67,69 284,292	10-11 77	7,10,62	12,61 79	20,72,78 155-157
2	10	11-13	30, 39	10-13	16 82	3-4	13	20
3	2	9-11	--	10-13	16 82	3-4	13-14	20,22
4	4-5	32-33	6-7,14 30, 33	14-17	10	67-69	13-14	70-74 203-205
5	18	21-22	118	39-42	31-34	---	32	24-25
6	101	59	--	69-70	23 77	270-271	---	200-201
7	20	7-8 32-33	5, 14	18-19	46-47	94-96	141-144	---
8	18-19	34-36	--	93-97	---	12-14	12-13	24,26,82 262-263

UNIT II - FUNDAMENTAL CONCEPTS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
1	8,23,59,106 346,350	68	8,24,31 198	17,77,130-131 189,195 212-213	2,3,12-13 19,50,171	175-176
2	100-101	---	9-10	16	49-54 150-154	13
3	100	---	---	---	149,159 162,167	---
4	8-9	---	440-441 8,9	---	54	---
5	41-42 120	5-6	89	84	227-240	---
6	106-107	68	31	---	171,182	133
7	14-21	56	112,115,118 121,125 176-178	---	190-204	11
8	124-125	34-35	91-93	78-81	176-179	15

PERFORMANCE OBJECTIVE II-1

Identify the following:

- a) factor
- b) term
- c) coefficient
- d) exponent
- e) base
- f) power
- g) variable
- h) constant

a) The coefficient of x in $3x + 2y^2 + 6$ is:

A. 1

B. 2

C. 6

D. 3

Answer ____________________

b) Use the expression $3x^2 + 7$ to answer the following.

1. The coefficient is __________.
2. The exponent is __________.
3. The variable is __________.
4. The constant is __________.
5. The base is __________.
6. The power of the base is __________.

c) Which of the following expressions contains an exponent of 2?

A. $x + 2$

B. $2x$

C. x^2

D. None of the above

Answer ____________________

d) Which of the following expressions contains a coefficient of 1?

A. x

B. $x + 1$

C. x^3

D. All of the above

E. None of the above

Answer ____________________

PERFORMANCE OBJECTIVE II-2

Compute the value of a numerical expression using the order of operations rules.

a) Simplify the following expression
$20 \div 2 - 20 \div 10$.

A. -1

B. 8

C. 20

D. undefined

Answer ______________

b) Simplify the following expression
$28 + 0 \div 4 - 10 \times 2$.

A. 8

B. -6

C. 36

D. 13

Answer ______________

c) Give the first step in simplifying
$16 + 2 \times 10 + 13$.

A. Add 16 and 2.

B. Multiply 2 and 10.

C. Add 10 and 13.

D. None of the above

Answer ______________

d) Simplify the following:

1. $8 \div \frac{1}{2} + 3 \times 2$

   Answer ______________

2. $7 \times 2^3 - 2^2 \times 2 \div 4 + 2$

   Answer ______________

3. $\dfrac{2 + 3 \times 5^2}{12 \times 3 \div 4 + 2}$

   Answer ______________

PERFORMANCE OBJECTIVE II-3

Compute the value of a numerical expression punctuated with symbols of inclusion.

ASSESSMENT TASKS

a) Simplify 36 + 4[1 + (12-8) x 2].

- A. 72
- B. 76
- C. 360
- D. 400

Answer ____________

b) 4(7 + 9) means

- A. Multiply 4 times 7; then add 9
- B. Add 7 and 9; then multiply by 4
- C. Either of the above
- D. None of the above

Answer ____________

c) Insert symbols of inclusion to make each statement true.

1. 2 + 3 x 4 - 2 = 8

   Answer ____________

2. 5 + 6 - 4 x 3 = 11

   Answer ____________

3. 3 + 4 - 6 x 0 = 0

   Answer ____________

4. 6 + 5 x 3 - 7 ÷ 13 = 2

   Answer ____________

d) Simplify.

1. [9 x (11 + 3)] ÷ 60

   Answer ____________

2. 13 + [99 - (13 x 7)]

   Answer ____________

3. $\dfrac{(5 \times 7) + (40 \div 2)}{7 + (16 \div 4)}$

   Answer ____________

4. $5\left\{\dfrac{[4 \times (36 \div 3)] + 2}{[(56 \div 7) \times 3] + 1}\right\}$

   Answer ____________

PERFORMANCE OBJECTIVE II-4

Evaluate algebraic expressions when given numerical replacements for the variables.

a) Find the value of each expression if $a = 1$, $b = 2$, $c = 12$, and $d = 0$.

1. $5a + 3b$

   Answer ______________

2. $3b^3 + 3c$

   Answer ______________

3. $\left[(a + b)(b + c)\right]d$

   Answer ______________

4. $\frac{b + c}{2a - b}$

   Answer ______________

b) Find the value of each expression if $a = 1$, $b = 2$, $c = 3$, $x = 12$, $y = 0$, and $z = \frac{1}{2}$.

1. $\frac{cx - a}{bc + a}$

   Answer ______________

2. $cx(x - c)$

   Answer ______________

3. $(2b + 1)^2(a + c)$

   Answer ______________

4. $ay + x^2 - bz$

   Answer ______________

c) Find the perimeter of a rectangle whose length is 7.3 meters and width is 4.2 meters using the formula $P = 21 + 2w$.

Answer ______________

d) Use the following two formulae to find equivalent temperatures:

$F = 9/5\ C + 32°$

$C = 5/9\ (F - 32°)$

1. $C = 20°$, $F =$ __________

2. $F = 95°$, $C =$ __________

PERFORMANCE OBJECTIVE II-5

Place the proper symbol (=, <, >) between two numerical expressions to make a true statement.

Replace each ? with =, <, > to make each statement true.

a) $\frac{10 + 2}{2}$? 10 + 1

Answer ______________

b) 3 1/3 ? 2 + 1 3/9

Answer ______________

c) 1/2 ? 1/3

Answer ______________

d) 11 x (6 + 7) ? {5 x 5} x 6

Answer ______________

e) 50 ÷ (2 x 5) ? (16 ÷ 4) ÷ 2

Answer ______________

f) (9 + 45) ÷ (18 ÷ 3) ? (72 ÷ 9) - 3

Answer ______________

PERFORMANCE OBJECTIVE II-6

State the number of terms in a given algebraic expression.

State the number of terms in each expression.

a) $xy - wz$

Answer ______________

b) $\frac{4(3xy - 15)}{73}$

Answer ______________

Multiple-choice. Determine the number of terms in the following expressions.

c) $(x - y) + \frac{x}{bc}$

A. 1

B. 2

C. 3

D. 4

Answer ______________

d) $\frac{ab - bc}{xy + yz}$

A. 1

B. 2

C. 3

D. 4

Answer ______________

PERFORMANCE OBJECTIVE II-7

Translate word phrases into algebraic expressions.

a) Match each word phrase with the correct algebraic expression.

___ 1. sum of two numbers

___ 2. difference of two numbers

___ 3. product of two numbers

___ 4. quotient of two numbers

A. xy

B. p/q

C. $r - s$

D. $s + r$

b) Choose the open expression which describes the word phrase, "One less than twice the number of field goals."

A. $2n$

B. $2n + 1$

C. $2n - 1$

D. $2n < 1$

Answer ______________

For each word phrase, write an algebraic expression.

c) the sum of five and twice w

Answer ______________

d) three more than the product of seven and t

Answer ______________

e) the quotient of x and seven, decreased by five

Answer ______________

f) fifteen decreased by n

Answer ______________

g) 6 less than x

Answer ______________

PERFORMANCE OBJECTIVE II-8

Determine the solution set of an open sentence by replacing the variable with elements of a given domain.

Using the domain $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, find the solution set of the equation.

a) $2x + 1 = 11$

Answer _______________

b) $\frac{1}{2}x + 4 = 10$

Answer _______________

c) $2x - 1 < 7$

Answer _______________

d) $2x + 3 \le x + 4$

Answer _______________

UNIT II - FUNDAMENTAL CONCEPTS

Answers

1. a) D

b) 1. 3

2. 2

3. x

4. 7

5. x

6. 2

c) C

d) D

2. a) B

b) A

c) B

d) 1. 22

2. 56

3. 7

3. a) A

b) B

c) 1. $2 + 3(4 - 2) = 8$

2. $5 + \left[(6 - 4) \times 3\right] = 11$

3. $(3 + 4 - 6) \times 0 = 0$

4. $\left[(6 + 5) \times 3 - 7\right] \div 13 = 2$

d) 1. 2.1

2. 21

3. 5

4. 10

4. a) 1. 11

2. 60

3. 0

4. undefined

b) 1. 5

2. 324

3. 100

4. 143

c) 23m

d) 1. 68°

2. 35°

5. a) $<$

b) $=$

c) $>$

d) $<$

e) $>$

f) $>$

6. a) 2

b) 1

c) B

d) A

7. a) 1. D

2. C

3. A

4. B

b) C

c) $5 + 2w$

d) $7t + 3$

e) $\frac{x}{7} - 5$

f) $15 - n$

g) $x - 6$

8. a) 5

b) $\emptyset$

c) 1, 2, 3

d) 1

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

PURPOSE

Unit III provides the students with an introduction to the rules and properties that govern the language of algebra under the system of real numbers.

These properties are integrated with the basic operations on directed numbers to prepare the student to solve open sentences. These ideas add to the base upon which the course will be built.

OVERVIEW

A working knowledge of the basic properties of real numbers is an expected outcome. In a minimal or average course, emphasis should be placed on the applications of these properties; while in a maximal course, a more comprehensive understanding of the theory should be stressed.

Computations with directed numbers are reviewed, and the concept of absolute value is introduced. It is mandatory that all students demonstrate minimal competencies in working with directed numbers before proceeding to the next unit.

SUGGESTIONS FOR THE TEACHER

Instructional Days: 8-9
Minimal Course Objectives: #4-11, 13
Average Course Objectives: ALL
Maximal Course Objectives: ALL

Most texts relate real numbers to the horizontal number line. The concept of operations on positive and negative numbers can also be illustrated by examples dealing with up and down, gain (profit) and loss, right and left, above and below, north and south, east and west, or good and bad.

The term directed number is used in this unit instead of real number to emphasize the positive-negative aspects without restricting problems to integers or involving irrational numbers.

The intention of Objective 3 (identify irrational numbers) is that the student realize there exist some real numbers which are not integers, fractions, terminating decimals, or repeating decimals and that students will be able to give some examples of irrationals, such as $\sqrt{2}$, π, .12345678910, ...

It is important to stress absolute value and its use in addition and subtraction of directed numbers.

Some tricks to help students remember the rules for multiplying and dividing directed numbers may be useful. One such trick involves using the idea of "good" to represent positive numbers and "bad" to represent negative numbers. Hence, the product of a negative number and a positive number could be remembered by the saying: "If bad things happen to a good person, that's bad." Similar sayings are possible for the other permutations.

It is important to realize that the subtraction sign, negative sign, opposite sign, and opposite of a sum often are difficult concepts for students to understand.

Fractions and decimals can be reviewed by including them in practice problems.

VOCABULARY

absolute value
additive identity
additive inverse (opposite)
associative property of addition
associative property of
   multiplication
axiom
closure property of real numbers

commutative property of
   multiplication

distributive property of
   multiplication with respect
   to addition

integers (Z)
irrational numbers (Ir)
like terms
magnitude
multiplicative identity

multiplicative inverse (reciprocal)
multiplicative property of zero
non-terminating, non-repreating
   decimals
postulate
property
rational numbers (Q)
real numbers
reflexive property of equality
repeating decimals
similar terms
substitution principle
symmetric property of equality
terminating decimals
transitive property of equality

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

ENTERING PERFORMANCE OBJECTIVES

1. Add, subtract, multiply, and divide directed numbers.
2. Evaluate algebraic expressions.

Assessment Tasks

1. a) $-11 + 5$ a) ______________

b) $9 + (-3)$ b) ______________

c) $-7 + (-11)$ c) ______________

d) $-5 + (-32)$ d) ______________

e) $-6\frac{1}{2} - (-3\frac{1}{2})$ e) ______________

f) $7\frac{1}{2} - 9\frac{1}{2}$ f) ______________

g) $300 - (-200)$ g) ______________

h) $3 - 12$ h) ______________

i) $(-0.25)\ (-8)$ i) ______________

j) $-7\ (21)$ j) ______________

k) $(-2)\ (3\frac{1}{2})$ k) ______________

l) $(-2)\ (-5)\ (-3)$ l) ______________

m) $18 \div (-2)$ m) ______________

n) $(-26) \div (-8)$ n) ______________

o) $(-12) \div \frac{1}{2}$ o) ______________

p) $(-\frac{5}{9}) \div (-\frac{9}{5})$ p) ______________

2. If $a = -5$, $n = 7$, $t = 10$ and $m = -3$, find the value of each of the following:

a) $\frac{a + t + 4}{m} + n$ a) ______________

b) $\frac{a^2 + n^2 + 1}{a^2}$ b) ______________

c) $\frac{a - m^2}{-2n^2}$ c) ______________

d) $\frac{1}{2}mt^2$ d) ______________

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) - 6

   b) 6

   c) - 18

   d) - 37

   e) 3

   f) - 2

   g) 500

   h) - 9

   i) 2

   j) -147

   k) - 7

   l) - 30

   m - 9

   n) $3\frac{1}{4}$

   o) - 24

   p) $\frac{25}{81}$

2. a) 4

   b) 3

   c) $\frac{1}{7}$

   d) -150

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

PERFORMANCE OBJECTIVES

1. Determine whether a given set of numbers is closed under a given operation. (III)

2. Identify the following properties:

    a) reflexive property of equality

    b) symmetric property of equality

    c) transitive property of equality

    d) commutative (+)

    e) commutative (x)

    f) associative (+)

    g) associative (x)

    h) distributive property of multiplication with respect to addition

    i) additive identity

    j) multiplicative identity

    k) additive inverse

    l) multiplicative inverse

    m) multiplicative property of zero (I)

    n) substitution principle

3. Distinguish between rational and irrational numbers. (III)

4. State the absolute value of a given real number. (II)

5. Add directed numbers. (II)

6. State the additive inverse (opposite) of a given real number. (II)

7. Subtract directed numbers. (II)

8. Multiply directed numbers. (II)

9. State the multiplicative inverse (reciprocal) of a given real number. (II)

10. Divide directed numbers. (II)

11. Simplify mathematical expressions involving directed numbers. (II)

12. Simplify numerical expressions containing absolute values. (II)

13. Simplify algebraic expressions by combining similar (like) terms. (II)

Minimum	Average	Maximal
#4 - 11, 13	ALL	ALL

KEY SKILLS FOR END-OF-COURSE TESTING

4. Add and subtract directed numbers.

5. Multiply and divide directed numbers.

6. Combine similar terms.

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	88 376	52-54	--	49-50 56-59	470-472	29-31 448-452	19-20	34
2	67,88-94 180	52-61 71-74 98-105	38, 40, 47 - 49	52-56 61-66 77	19-21 65,70 500	26-28 41,56 60,113	122 19-24,28-30 33-34,40-41	34-36, 56 115-116 184
3	386-390	401-406	335	30-33	389-393	22	40-44	13-16
4	39-40	76-79	63, 89	25-28	54-56	42	69-70	17 138-40
5	42-45	64-75, 92-97	58 - 67	43-51	57-60	45-47	73-76	30-33 38-40
6	38-40	71-75 92-97	--	52-55	57-60	41-42	33-34 67-68	34
7	52-55	119-126	69 - 72	72-76	64-67	51-53	86-88	42-45
8	63-65	98-107	89 - 90	56-65	68-72	55-56	77-81	46-49
9	70-72	104-107	91 - 93	77-81	73-75	60	38-39	56
10	78-81	130-136	--	77-81	73-75	60-61	88-90	50-52
11	55, 68-69	101	49 - 50	60,86	---	---	82	66 68-70
12	40	76-79	--	25-28	---	43	78	17-18
13	101-103	58-63	49 - 50	64-71	76-79	64-69	61-64	78-80

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
1	157-160	---	---	29,33	---	---
2	154-157 267-269 162-168,173 246-249	13-15, 7-8 22, 25	20-23 26,33	29-42	104-106 112-116	---
3	---	342-343	355-356	27	47	260
4	261	10-11, 16	228	2	128,136	30
5	236-240	16-18	234-242	---	146,149	---
6	240-245	7-8	218	29	136	---
7	250-252	19-21	294-298	---	128-146	---
8	264-266 270-272	22-23	266-271	---	128-146	---
9	90 278-281	25-26	70-71 308	33	128-146	---
10	276-277	25-27	311-312	---	128-146	---
11	252, 272, 277,268-269	23	---	---	128-146	---
12	261	11	229	2-3	107-109	---
13	273-275	38-40	31,126 284,301	189-191	107-109	21

PERFORMANCE OBJECTIVE III-1

Determine whether a given set of numbers is closed under a given operation.

Determine whether the given set is closed under the given operation.

_____ a) $\{1, 2\}$, addition

_____ b) $\{0, 1\}$, multiplication

_____ c) $\{2, 4, 6, \ldots\}$, division

Under which operation(s) (+, -, x, ÷) are the following sets closed?

_____ d) $\{0, 2, 4, 6\}$

_____ e) $\{3, 6, 9, 12 \ldots\}$

_____ f) {Integers}

PERFORMANCE OBJECTIVE III-2

Identify the following properties: reflexive, symmetric, and transitive; commutative property of addition, commutative property of multiplication; associative property of addition, associative property of multiplication; distributive property of multiplication with respect to addition; additive inverse, multiplicative inverse; multiplicative property of zero; substitution principle; additive identity, multiplicative identity.

a) Match each property with the appropriate example:

_____ 1. If $5 + 0 = 5$, then $5 = 5 + 0$

_____ 2. $5(x + y) = 5x + 5y$

_____ 3. $(r + 3)\ 19 = 19\ (r + 3)$

_____ 4. $x + (7 + 6x) = x + (6x + 7)$

_____ 5. $97\ (100 + 1) = 97\ (101)$

A. Substitution principle

B. Commutative property of addition

C. Commutative property of multiplication

D. Symmetric property of equality

E. Distributive property of multiplication with respect to addition

b) State algebraically the following properties:

1. Associative property of addition

____

2. Transitive property of equality

____

3. Multiplicative identity

____

PERFORMANCE OBJECTIVE III-2 (continued)

Identify the following properties: reflexive, symmetric, and transitive; commutative property of addition, commutative property of multiplication; associative property of addition, associative property of multiplication; distributive property of multiplication with respect to addition; additive inverse, multiplicative inverse; multiplicative property of zero; substitution principle; additive identity, multiplicative identity.

c) Name the property which justifies each step in the following:

1. $16 + (27 + 14) = 16 + (14 + 27)$ ________________

2. $16 + (14 + 27 \quad (16 + 14) + 27$ ________________

3. $(16 + 14) + 27 = 30 + 27$ ________________

4. $30 + 27 = 57$ ________________

d) For all real numbers a and b:

1. $(a + 3) + (5 + b) = \left[(a + 3) + 5\right] + b$ ________________

2. $= \left[a + (3 + 5)\right] + b$ ________________

3. $= (a + 8) + b$ ________________

4. $= (8 + a) + b$ ________________

5. $= 8 + (a + b)$ ________________

PERFORMANCE OBJECTIVE III-3

Distinguish between rational and irrational numbers.

a) True or false ?

____ 1. $\{\text{irrational numbers}\} \subset \{\text{rational numbers}\}$

____ 2. $\{\text{irrational numbers}\} \subset \{\text{real numbers}\}$

____ 3. $\{0\} \subset \{\text{irrational numbers}\}$

____ 4. $\{\text{integers}\} \subset \{\text{irrational numbers}\}$

Identify each number as rational (Q) or irrational (Ir).

____ b) 0

____ c) $\sqrt{2}$

____ d) $.1\overline{6}$

____ e) .12112111211112...

PERFORMANCE OBJECTIVE III-4

State the absolute value of a given real number,

Simplify:

a) $|-28.5| =$ __________

b) $|0| =$ __________

c) $|16| =$ __________

d) True or false?

_____ 1. The absolute value of a negative number is positive,

_____ 2. The absolute value of a positive number is negative.

_____ 3. The absolute value of 0 is 0,

_____ 4. The absolute value of a positive number is positive,

PERFORMANCE OBJECTIVE III-5

Add directed numbers.

a) Add each of the following:

1. $\begin{array}{r} -17 \\ \underline{+\ 8} \end{array}$ 2. $\begin{array}{r} -28 \\ \underline{-36} \end{array}$ 3. $\begin{array}{r} -28 \\ \underline{0} \end{array}$ 4. $\begin{array}{r} -15 \\ \underline{+42} \end{array}$

b) Add each of the following:

_____ 1. +27 + (-9 + -13)

_____ 2. -7.5 + 8.8

_____ 3. $-5\frac{1}{4} + 8\frac{1}{2}$

_____ 4. -6 + -4

c) True or false?

_____ 1. -6 + -9 = 15

_____ 2. 9 + -3 > -6

_____ 3. (-3 + 4) + -1 = -3 + (4 + -1)

_____ 4. 5 + -3 > -5 + 3

d) Add the following:

_____ 1. -98 + (-2 + 53)

_____ 2. -101 + (-99 + 28)

_____ 3. (19 + -16) + (11 + -23)

_____ 4. (-1.2 + -5.8) + 4

PERFORMANCE OBJECTIVE III-6

State the additive inverse of a given real number.

Give the opposite of each number:

a) 1. 13 ________

2. -16 ________

b) 1. 0 ________

2. $-1\frac{1}{3}$ ________

c) 1. a ________

2. -b ________

d) 1. $\frac{1}{3}$ ________

2. $-\frac{1}{2}$ ________

PERFORMANCE OBJECTIVE III-7

Subtract directed numbers.

a) Match each difference with the corresponding sum:

_____ 1. 9 - 6

_____ 2. -9 - 6

_____ 3. -9 - -6

_____ 4. 9 - -6

A. -9 + 6

B. -9 + -6

C. 9 + -6

D. 9 + 6

Subtract:

Answers

b) -45 - +17 _____

c) -38 - (-12) _____

d) +15 - +18 _____

PERFORMANCE OBJECTIVE III-8

Multiply directed numbers.

a) Find the value for x that makes each statement true.

_____ 1. $-3x = 9$

_____ 2. $-(x) = 4$

_____ 3. $-2x\ (-1) = 10$

_____ 4. $x(-3)(-2) = -30$

Multiply the following:

Answers

b) 1. $(-13)(+3) =$ _____

2. $(-6)(0) =$ _____

c) 1. $-4(-3) =$ _____

2. $-6(2) =$ _____

d) 1. $(-4)3 =$ _____

2. $2(-2)=$ _____

PERFORMANCE OBJECTIVE III-9

State the multiplicative inverse of a given real number.

State the multiplicative inverse for each of the following:

Answers

a) 1. 1 ______

2. -1 ______

b) 1. $\frac{5}{3}$ ______

2. $-\frac{7}{-2}$ ______

c) 1. $\frac{1}{2}$ ______

2. 0 ______

d) 1. $3\frac{1}{4}$ ______

2. -2.4 ______

PERFORMANCE OBJECTIVE III-10

Divide directed numbers.

a) Write each quotient as a product:

Answer

1. $-36 \div -2$ ________
2. $36 \div -\frac{1}{2}$ ________
3. $\frac{14}{-\frac{2}{3}}$ ________
4. $\frac{\frac{1}{2}}{5}$ ________

Divide each of the following:

b) $-144 \div {}^{+}9$ ________

c) $-75 \div {}^{-}15$ ________

d) ${}^{+}100 \div {}^{-}20$ ________

PERFORMANCE OBJECTIVE III-11

Simplify mathematical expressions involving directed numbers,

Let $a = 2$, $b = -7$, $c = 6$, $d = -5$, $e = 4$ and $f = -1$. Evaluate each of the following expressions.

_____ a) $abc - def$

_____ b) $\frac{abc}{e}$

_____ c) $\frac{ab + cd}{ef}$

_____ d) $\frac{af - de}{b + c}$

PERFORMANCE OBJECTIVE III-12

Simplify numerical expressions containing absolute values,

Simplify both a) and b).	Answers:
a) 1. $-\lvert -9 \rvert$	a) 1. ______________
2. $\lvert - \lvert -7 \rvert\rvert$	2. ______________
3. $- \lvert 0 \rvert$	3. ______________
4. $-(\lvert {}^{-}\lvert -6 \rvert\rvert)$	4. ______________
b) 1. $7 \lvert -6 \rvert$	b) 1. ______________
2. $3 \lvert -4 \rvert + \lvert -3 \rvert$	2. ______________
3. $-1 \lvert -11 + 7 \rvert$	3. ______________
4. $4 \lvert -7 \rvert + \lvert -12 \rvert$	4. ______________

c) State whether x or y represents the greater real number.

_____ 1. The absolute value of x is greater than the absolute value of y. Both x and y are positive.

_____ 2. The absolute value of x is less than the absolute value of y. Both x and y are negative.

_____ 3. The absolute value of x is greater than the absolute value of y. Both x and y are negative.

d) Choose two numbers, m and n, such that the absolute value of m is greater than the absolute value of n and m is less than n.

Answer______________

PERFORMANCE OBJECTIVE III-13

Simplify algebraic expressions by combining similar terms,

Simplify by combining similar terms.

Answers

a) 1. $9r - 5r$ __________

2. $-6m + m$ __________

b) 1. $8x + 5y - 3x + y$ __________

2. $-7x + 7 + 3x - 3$ __________

c) 1. $-3r + 3s + 2r - 2s$ __________

2. $-2x + 2(2x - 4)$ __________

d) 1. $-r - r - 3r$ __________

2. $4r - 3(r + 1) + 3$ __________

UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

Answers

1. a) not closed
    b) closed
    c) not closed
    d) none
    e) +, x
    f) +, -, x

2. a) 1. D
        2. E
        3. C
        4. B
        5. A

    b) 1. $x + (y + z) = (x + y) + z$
        2. If $a = b$ and $b = c$ then $a = c$
        3. $1(x) = x$
    c) 1. Commutative (+)
        2. Associative (+)
        3. Substitution
        4. Substitution

    d) 1. Associative (+)
        2. Associative (+)
        3. Substitution
        4. Commutative (+)
        5. Associative (+)

3. a) 1. F
        2. T
        3. F
        4. F
    b) Q
    c) Ir
    d) Q
    e) Ir

4. a) 28.5
    b) 0
    c) 16
    d) 1. T
        2. F
        3. T
        4. T

5. a) 1. -9
        2. -64
        3. -28
        4. +27
    b) 1. +5
        2. 1.3
        3. $3\frac{1}{4}$
        4. -10

Answers (continued)

5. c) 1. F
2. T
3. T
4. T
d) 1. -47
2. -172
3. -9
4. -3.0
6. a) 1. -13
2. 16
b) 1. 0
2. $1\frac{1}{3}$
c) 1. -a
2. b
d) 1. $-\frac{1}{3}$
2. $\frac{1}{2}$
7. a) 1. C
2. B
3. A
4. D
b) -62
c) -26
d) -3

8. a) 1. -3
2. -4
3. 5
4. -5
b) 1. -39
2. 0
c) 1. 12
2. -12
d) 1. -12
2. -4
9. a) 1. 1
2. -1
b) 1. $\frac{3}{5}$
2. $\frac{2}{7}$
c) 1. 2
2. No reciprocal
d) 1. $\frac{4}{13}$
2. $-\frac{5}{12}$
10. a) 1. $-36 \times -\frac{1}{2}$
2. 36×-2
3. $14 \times \frac{3}{-2}$
4. $\frac{1}{2} \times \frac{1}{5}$
b) -16
c) 5
d) -5

11. a) -104
b) -21
c) 11
d) -18

12. a) 1. -9
2. 7
3. 0
4. -6
b) 1. 42
2. 15
3. -4
4. 40
c) 1. x
2. x
3. y
d) Answer varies
13. a) 1. 4r
2. -5m
b) 1. 5x + 6y
2. -4x + 4
c) 1. -r + s
2. 2x - 8
d) 1. -5r
2. r

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

PURPOSE

This unit provides students with the opportunity to apply the skills mastered in previous units. Emphasis is on the techniques for solving open sentences. Students are introduced to equations and inequalities that require a sequence of transformations. A systematic approach to work problem analysis and solution is developed. This unit is the keystone of the course.

OVERVIEW

Basic transformations for solving open sentences are presented. They are applied to equations of varying complexity.

Inequalities are treated in a similar manner. Emphasis is placed on the difference between the multiplication and division transformations for equality and the corressponding transformations for inequality.

Specific techniques for analyzing and solving word problems are discussed. These techniques are applied to the solution of consecutive integer, age, geometry, uniform motion, and mixture problems.

SUGGESTIONS TO THE TEACHER

Instructional Days:	20-25
Minimal Course Objectives:	#1-8, 10, 12-15
Average Course Objectives:	ALL except #11
Maximal Course Objectives:	ALL

Due to the length of this unit, it is suggested that at least two tests be given. Emphasize to students that transformed equations are always equivalent to the original equations. They have the same solution set.

Students frequently have difficulty in deciding what order to apply transformations to solve equations of the form $ax + b = c$. A technique to use in explaining the solution of these is the following analogy to unwrapping a birthday present. In opening a birthday present that is wrapped in paper and tied with a ribbon, one could remove either first. However, it would be much easier to remove the ribbon prior to removing the paper. In like manner, in equations of the form $ax + b = c$, it is frequently easier to remove the constant term before removing the coefficient.

Flow charts can be very useful in teaching the procedure for solving equations. It is strongly suggested that a check of all solutions be required.

The importance of proficiency in the reading of word problems cannot be overemphasized. The SQ3R method (survey, question, read, recite, and review) can be a valuable approcah to use.

Objectives #12-15 suggest an approach for attacking word problems. Teachers may want to use their own strategies, and suggestions would be welcomed by the math coordinator.

The use of calculators may be desirable to facilitate computations in the solution of word problems, particularly when decimals are involved.

VOCABULARY

addition property of equality
addition property of inequality
angle
comparison property
complementary angles
conjunction
consecutive integers
degree
disjunction
distance
division property of equality
division property of inequality
equation
equivalent equation
equivalent inequality
inequality
left member
linear equation
multiplication property of equality
multiplication property of inequality
open sentence
rate
ray
right member
root
solution set
subtraction property of equality
subtraction property of inequality
supplementary angles
transformations
transition property of inequality

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

ENTERING PERFORMANCE OBJECTIVES

1. Add, subtract, multiply, and divide directed numbers.
2. Name the opposite and reciprocal of a number.
3. Combine similar terms.
4. Graph a set on a number line.
5. Multiply algebraic expressions using the distributive property.

Assessment Tasks

1. a) $(-2\frac{1}{3}) + 2$ a) ____________________

b) $(-8) + 4\frac{1}{2}$ b) ____________________

c) $143 + (-482)$ c) ____________________

d) $-6.4 + 16.6 + (-12.9) + 8.4$ d) ____________________

e) $-0.6 - 0.4$ e) ____________________

f) $-0.50 - (-0.20)$ f) ____________________

g) $-10 - 32$ g) ____________________

h) $12 - [-(5 + 3)]$ h) ____________________

i) $-3(0.321)$ i) ____________________

j) $(-\frac{1}{5})(\frac{3}{4})$ j) ____________________

k) $(-3)(-5)(2)$ k) ____________________

l) $(-0.5)^3$ l) ____________________

m) $3.5 \div (-.7)$ m) ____________________

n) $-3\frac{3}{5} \div (-\frac{9}{10})$ n) ____________________

p) $108 \div (-6)$ p) ____________________

q) $-.170 \div (-.05)$ q) ____________________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. a) State the opposite of each of the following numbers:

   (1) -5

   (2) 6.37

   (3) -4

   (4) 0

   b) State the reciprocal of each of the following numbers:

   (1) 7

   (2) $-\frac{3}{4}$

   (3) $.7$

   (4) x

   c) Which of the following numbers is the reciprocal of 1.5?

   (1) -1.5

   (2) $\frac{3}{2}$

   (3) $\frac{2}{3}$

   (4) $1\frac{1}{2}$

   Answer ____________.

   d) If x is a negative number, which of the following is the opposite of x?

   (1) x

   (2) $\frac{1}{x}$

   (3) $-\frac{1}{x}$

   (4) $-x$

   Answer ____________.

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

3. Combine similar terms.

a) $(4 - 9t^2 + 15t^4) + (3t^4 + 2t^2 - 1)$ a) ________________

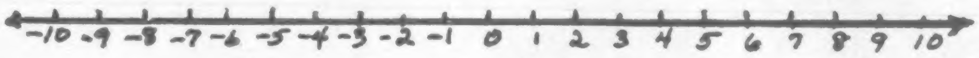
b) $(8x^3 - x^2 - 5x + 6) + (14x^3 - 7x^2 + 10x + 15)$ b) ________________

c) $(17m^2 - 13m - 6) - (3 + 7m + 17m^2)$ c) ________________

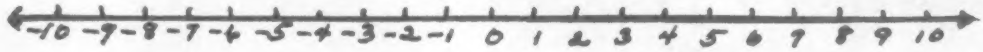
d) $(2x + 7) - (\frac{1}{2}x + 9)$ d) ________________

4. Graph the following sets on a horizontal number line:

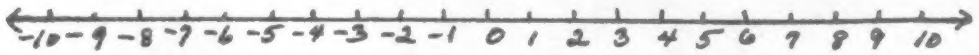
a) {whole numbers less than 8}


-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

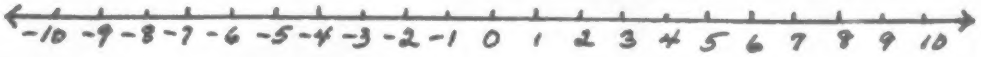
b) {integers between -5 and 3}


-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

c) {real numbers greater than -2}


-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

d) {real numbers between -7 and 3, including 3}


-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

5. Simplify each expression.

a) $7(x^2 + y) + 3(x^2 + y) + 9y$ a) ________________

b) $8(2x^2 - 4x + 7) + 3(12x^2 - 5x)$ b) ________________

c) $2(3x^2 - 5x - 2) - 3(8 - 2x^2 + 7x)$ c) ________________

d) $-3(2m^2 - m + 4) - (2m^2 + 3m - 8)$ d) ________________

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

ENTERING PERFORMANCE OBJECTIVES

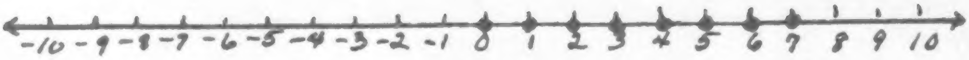
Answers

1. a) $-\frac{1}{3}$
   b) $-3\frac{1}{2}$
   c) -339
   d) 5.7
   e) -1.0
   f) -0.30
   g) -42
   h) 20
   i) -0.963
   j) $-\frac{3}{20}$
   k) 30
   l) -0.125
   m) -5
   n) 4
   p) -18
   q) 3.4

2. a) (1) 5
   (2) -6.37
   (3) 4
   (4) 0
   b) (1) $\frac{1}{7}$
   (2) $-\frac{4}{3}$
   (3) $\frac{10}{7}$
   (4) $\frac{1}{x}$
   c) (3)
   d) (4)

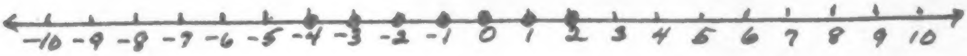
3. a) $18t^4 - 7t^2 + 3$
   b) $22x^3 - 8x^2 + 5x + 21$
   c) $-20m - 9$
   d) $\frac{3}{2}x - 2$

4. a)



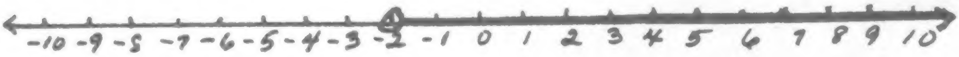
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

b)



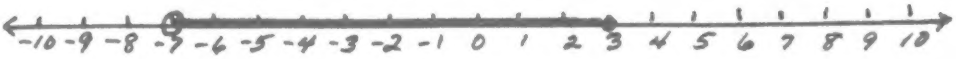
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

c)



-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

d)



-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

5. a) $10x^2 + 19y$
   b) $52x^2 - 47x + 56$
   c) $12x^2 - 31x - 28$
   d) $-8m^2 - 4$

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

PERFORMANCE OBJECTIVES

1. Identify the addition, subtraction, multiplication, and division properties of equality. (I)

2. Solve equations of the form $x + a = c$, using the addition (subtraction) property of equality. (III)

3. Solve equations of the form $ax = c$, using the multiplication (division) property of equality. (III)

4. Solve equations of the form $ax + b = c$, using the appropriate properties. (III)

5. Solve equations of the form $ax + b = cx + d$, during the appropriate properties. (III)

6. Solve equations involving the distributive property of multiplication with respect to addition. (III)

7. Identify the addition, subtraction, multiplication, and division properties of inequality. (I)

8. Solve inequalities, using the appropriate properties. (III)

9. Solve compound inequalities by applying the concepts of union and intersection of sets. (III)

10. Graph the solution set of an inequality or compound inequality on the real number line. (III)

11. Solve open sentences involving absolute value. (III)

12. Translate word statements into open sentences. (III)

13. Write algebraic expressions representing the unknown information in a word problem. (IV)

14. Write an open sentence expressing the relationship stated in a problem. (IV)

15. Solve various types of word problems, utilizing an organized approach. (IV)

Minimal	Average	Maximal
#1 - 8, 10, 12 - 15	All	All

KEY SKILLS FOR END-OF-COURSE TESTING

7. Solve linear equations in one variable applying the properties of equality.

8. Solve linear inequalities in one variable applying the properties of inequality.

9. Solve various types of word problems utilizing an organized approach.

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	48,74	117,127	73 94, 98	99,107	102,104	76,79	123,129-130	82,85
2	48-49,56 56-57	119	73-77	101-102	102-109	79-81	123-124	83,84
3	74-77 82-83	129	94-99	110	104-105	76-78	129-131	86,87
4	85-87	138-139	102-104	111-114	106-109	82-84	130-131	93-95
5	107-109	148-150	--	115-118	107-109	88	130-131 127	96-97
6	104-106	139	--	118	110-111	89,93	130-132 127	96-97
7	261-264	162-163	124-129	119-127	276-278, 279-281	92	125,132	118 122
8	261-265	166-167	124-131	126,130	276-283	93	125-127 132-135	120,124 121
9	274-278	168-174	132-135	134,138	---	107-110	135-138	134-137
10	274-278	167	132-135	131-132	284-286	90-93	135-138	135-137
11	280-281	174-176	137-139	---	285-286	110-112	135-138	138-140
12	20-24	140-143	23-24 52 100-101	143-148	46-47 93-95 116-122	97	141-144	141-144
13	20-24, 110 104-105,267- 113-114,268	144-145	51-52 97 100-101	145-148	46-47 93-95 116-122	96	145-151	98-99
14	20-24,87,110 104-105,267- 113-114 268	144-145	106-108 110-112	146-148	46-49 93-95 116-122	96	145-151	143
15	87,104-105 113-115 109,110-111	146-148 151-153 176-192	97 100-112	.49-157	46-49 93-95 116-122	102-106	145-151	88-91 98-102

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
1	186	44-49	110,113 116,119	52-55	172-173	52-55
2	186-188	44-46	110,113	52-54	175-176	52-54
3	189-193	47-49	116-119	55-57	176-179	55-57
4	194-196	52-55	122-124	58-60	176-179	58-60
5	201-202	57-58	334	64-66	187-194	64-66
6	354	85	126-127	62-63	187-194	62-63
7	305-306 308-309	65,92-93 280	---	82-86 90-91	293	82-86
8	305-306 308-309	280-284	---	85-95	240	90-91
9	---	---	---	---	251	85-95
10	143-145	282-284	---	---	236,242	---
11	---	---	---	---	251-252	---
12	198-199	56	176	67-71 296-315 444-446	253	67-71 296-315 444-446
13	198-200	---	176	---	91-94	---
14	---	---	176	---	180-181	---
15	---	---	180-198	---	185 194-195	---

PERFORMANCE OBJECTIVE IV-1

Identify the addition, subtraction, multiplication and division properties of equality.

Identify the property of equality that justifies the following transformations.

a) 1) If $a + x = y$ then $a + x + 5 = y + 5$

Answer ______________

2) If $a = b$ then $-3a = -3b$

Answer ______________

b) 1) If $x = y$ then $x + a = y + a$

Answer ______________

2) If $a + x = b$ then $\frac{1}{2}a + \frac{1}{2}x = \frac{1}{2}b$

Answer ______________

c) Match the following properties of equality with their respective names.

___ 1) If $a = b$ then $ac = bc$

___ 2) If $a = b$ then $a + c = b + c$

___ 3) If $a = b$ then $a/c = b/c$

___ 4) If $a = b$ then $a - c = b - c$

A. Addition Property

B. Subtraction Property

C. Multiplication Property

D. Division Property

d) State the property that justifies each step.

Given: $5x - 3 = 2x + 6$

1) $5x - 2x - 3 = 2x - 2x + 6$

Answer ______________

2) $3x - 3 = 6$

Answer ______________

3) $3x - 3 + 3 = 6 + 3$

Answer ______________

4) $3x = 9$

Answer ______________

5) $\frac{3x}{3} = \frac{9}{3}$

Answer ______________

6) $x = 3$

Answer ______________

PERFORMANCE OBJECTIVE IV-2

Solve equations of the form $x + a = c$ using the addition (subtraction) property of equality.

a) Identify the following pairs of equations as equivalent or not equivalent.

__________ 1) $x + 2 = 7$
$x + 3 = 8$

__________ 2) $y = -4$
$7 + y = 3$

__________ 3) $-2 + x = 8$
$x = 6$

__________ 4) $5 + m = 19$
$(5 + m) + 2 = 19 + 2$

Solve each equation. Show all work including a check.

b) $m + 7 = 13$ Answer:

c) $x + 3\frac{1}{2} = 7$ Answer:

d) $3.7 = a + 11.2$ Answer:

PERFORMANCE OBJECTIVE IV-3

Solve equations of the form $ax = c$, using the multiplication (division) property of equality.

a) State what number each phrase must be multiplied by to get x as the result.

________ 1) $3x$

________ 2) $-\frac{1}{2}x$

________ 3) ax

________ 4) $\frac{2x}{3}$

Solve each equation. Show all work including a check.

b) $13t = -52$ Answer:

c) $-8 = -1/5\, b$ Answer:

d) $-14k = 84$ Answer:

PERFORMANCE OBJECTIVE IV-4

Solve equations of the form ax + b = c, using the appropriate properties.

a) Number the following steps in correct order to solve 4x-5 = 3.

___ A. $\frac{4x}{4} = \frac{8}{4}$

___ B. 4x - 5 + 5 = 3 + 5

___ C. x = 2

___ D. 4x = 8

b) Given the equation 3x + 4 = 7, the most efficient first step in the solution would be:

A. Multiply each side by 1/3

B. Divide both sides by 3

C. Divide 3x and 7 by 3

D. Subtract 4 from each side

Answer ______________

Solve each equation. Show all work including a check.

c) 5x - 9 = -19 Answer:

d) 8 = 5x + 33 Answer:

PERFORMANCE OBJECTIVE IV-5

Solve equations of the form $ax + b = cx + d$, using the appropriate properties.

a) Tell what phrase you would add to each side of the equation to eliminate the variable from the right side.

________ 1) $2x + 3 = 4x - 1$

________ 2) $3 + 5x = 5 - 2x$

________ 3) $x + 7 = 2 - \frac{1}{2}x$

________ 4) $7 - 8x = x + 2$

Solve each equation. Show all work including a check.

b) $7x + 5 = 2x + 35$ Answer:

c) $7y - 5 = 9y + 29$ Answer:

d) $16 + 4y = 10y - 20$ Answer:

PERFORMANCE OBJECTIVE IV-6

Solve equations involving the distributive property of multiplication with respect to addition.

Solve each equation. Show all work including a check.

a) $5(3y - 2) = 5$ Answer:

b) $7(x + 2) = 5(x + 4)$ Answer:

c) $3d + 2(6d - 5) = 5$ Answer:

d) $3(x + 2) + 3x + 2 = 74$ Answer:

PERFORMANCE OBJECTIVE IV-7

Identify the addition, subtraction, multiplication, and division properties of inequality.

a) Identify each pair of inequalities as equivalent or not equivalent.

1. $x + 5 < m$

   $x > m - 5$

   Answer ______________

2. $5x < m$

   $x < m/5$

   Answer ______________

3. $-5x < m$

   $x < \frac{-1m}{5}$

   Answer ______________

4. $\frac{x}{5} < m$

   $x < 5m$

   Answer ______________

b) Match the following properties of inequality with their respective names.

____ 1. Addition Property

____ 2. Subtraction Property

____ 3. Multiplication Property

____ 4. Division Property

In the following statements $a, b, c \in R$.

A. If $a < b$ and $c > 0$, then $a/c < b/c$
   If $a < b$ and $c < 0$, then $a/c > b/c$

B. If $a < b$, then $a + c < b + c$

C. If $a < b$ and $c > 0$, then $ac < bc$
   If $a < b$ and $c < 0$, then $ac > bc$

D. If $a < b$, then $a - c < b - c$

c) Explain the difference between the multiplication property of equality and the multiplication property of inequality.

Answer:

d) State the property which justifies each step.

Given: $6 - 3x \leq 9$

__________ 1. $6 - 6 - 3x \leq 9 - 6$

__________ 2. $-3x \leq 3$

__________ 3. $\frac{-3x}{-3} \geq \frac{3}{-3}$

__________ 4. $x \geq -1$

PERFORMANCE OBJECTIVE IV-8

Solve inequalities, using the appropriate properties.

a) In order to form pairs of equivalent sentences, tell whether < or > should be used as missing symbols.

_____ 1. $x < -3; \; -6 \; \underline{\;?\;} \; 2x$

_____ 2. $5y > 45; \; 9 \; \underline{\;?\;} \; y$

_____ 3. $\frac{1}{3} y < -7; \; y \; \underline{\;?\;} \; -21$

_____ 4. $-4a < 24; \; -6 \; \underline{\;?\;} \; a$

Solve each inequality. Show all work.

b) $b + 60 < -100$

Answer:

c) $-12x + 1 \geq 25$

Answer:

d) $16 - 4n \leq 6n - 24$

Answer:

PERFORMANCE OBJECTIVE IV-9

Solve compound inequalities by applying the concepts of union and intersection of sets.

Solve each open sentence. Show all work.

a) $x + 6 > 8$ and $x - 1 < 4$

Answer:

b) $x - 3 \leq -4$ or $x - 3 \geq 4$

Answer:

c) $-2 \leq x + 6 \leq 3$

Answer:

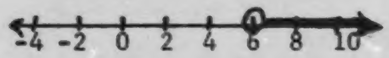
d) $y + 6 > 7$ or $3y + 5 < 3$

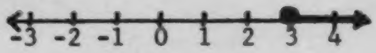
Answer:

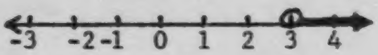
PERFORMANCE OBJECTIVE IV-10

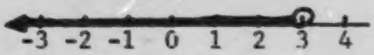
Graph the solution set of an inequality or compound inequality on the real number line.

a) Which of the following is the graph of $2x > 6$?

A.


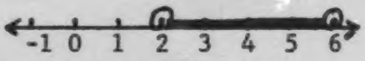
B.


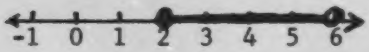
C.


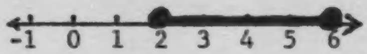
D.


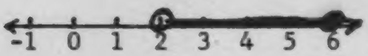
Answer ______________

b) Which of the following is the graph of $2 < y \leq 6$?

A.


B.


C.


D.


Answer ______________

c) Solve and graph the solution set of $6x + 2 - 8x < 14$

Answer:

d) Solve and graph the solution set of $5x \leq 10 + 2(3x - 4)$

Answer:

PERFORMANCE OBJECTIVE IV-11

Solve open sentences involving absolute value.

Solve each open sentence. Show all work.

a) $|2x + 3| = 11$

Answer:

b) $|x + 7| - 3 \geq 9$

Answer:

c) $5 + |2x - 5| \leq 7$

Answer:

d) $|3x + 4| < 13$

Answer:

PERFORMANCE OBJECTIVE IV-12

Translate word statements into open sentences.

a) Match the word phrase with its corresponding open sentence.

_____ 1. Twice a number is six. A. $x + y = xy$

_____ 2. Eight decreased by a number is 10. B. $x - 4 < 6$

_____ 3. The sum of two numbers is their product. C. $2x = 6$

_____ 4. 4 less than a number is less than 6. D. $8 - x = 10$

Translate each word statement into an open sentence.

b) Five times a number is 6 more than twice a number.

Answer _______________

c) Seven times a number is less than twice the sum of the number and seven.

Answer _______________

d) The product of 2 and 7 more than a number is equal to the sum of twice the number and 14.

Answer _______________

PERFORMANCE OBJECTIVE IV-13

Write algebraic expressions representing the unknown information in a word problem.

For each word problem answer the questions immediately following it.

a) Two cars started from the same point and traveled in opposite directions at rates of 30 km/h and 40 km/h. In how many hours will the cars be 2100 kilometers apart?

__________ 1. Identify a variable to represent what you are trying to find.

__________ 2. Write an expression to represent the distance traveled by the 30 km/h car.

__________ 3. Write an expression to represent the distance traveled by the 40 km/h car.

b) Find two consecutive integers whose sum is 57.

__________ 1. Identify a variable to represent the first integer.

__________ 2. In terms of this variable write an expression to represent the second integer.

c) Dr. D left his home by car, traveling on a certain road at the rate of 45 mph. Two hours later, his son Wes left home and started after him on the same road, traveling at the rate of 55 mph. Wes overtook his father in x hours.

__________ 1. Represent in terms of x the number of hours Dr. D traveled.

__________ 2. Represent in terms of x the distance Dr. D traveled.

__________ 3. Represent in terms of x the distance Wes traveled.

d) Find 3 consecutive even integers such that 4 times the first decreased by the second is 12 more than twice the third. Let x represent the first even integer.

__________ 1. In terms of x, represent the second even integer.

__________ 2. In terms of x, represent the third even integer.

PERFORMANCE OBJECTIVE IV-14

Write an open sentence expressing the relationship stated in a word problem.

For each of the following, write an algebraic sentence to solve the problem.

a) Find two consecutive integers whose sum is 57.

Answer ______________

b) Two cars started from the same point and traveled in opposite directions at rates of 30 mph and 40 mph. In how many hours will the cars be 2100 miles apart?

Answer ______________

c) How many pounds of $2.50 coffee and $2.87 coffee must a dealer mix to produce 70 pounds of coffee to sell for $2.61 per pound?

Answer ______________

d) Find the two greatest consecutive even integers whose sum is less than 80.

Answer ______________

PERFORMANCE OBJECTIVE IV-15

Solve various types of word problems utilizing an organized approach.

Solve each word problem. Show all work.

a) Barry owns one more than twice as many books as Elaine. If Barry owns 59 books, how many books does Elaine own?

Answer:

b) A football team won twice as many games as it lost. It won 18 games; how many did it lose?

Answer:

c) The school store sold 348 notebooks, some for 25 cents each and the rest at 38 cents each. The total receipts for notebooks was $100.91. How many of each kind was sold?

Answer:

d) At 8 a.m. two planes leave St. Louis. One flies west at 350 km/h. The other flies east at 400 km/h. At what time will they be 1500 kilometers apart?

Answer:

UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers

1. a) 1) Addition Property of Equality

   2) Multiplication Property of Equality

   b) 1) Addition Property of Equality

   2) Multiplication Property of Equality

   c) 1) C

   2) A

   3) D

   4) B

   d) 1) Subtraction Property of Equality

   2) Substitution

   3) Addition Property of Equality

   4) Substitution

   5) Division Property of Equality

   6) Substitution

2. a) 1) Equivalent

   2) Equivalent

   3) Not Equivalent

   4) Equivalent

   b) $m + 7 = 13$
   $m + 7 - 7 = 13 - 7$
   $m = 6$
   Check: $6 + 7 = 13$
   $13 = 13$

   c) $x + 3\frac{1}{2} = 7$
   $x + 3\frac{1}{2} - 3\frac{1}{2} = 7 - 3\frac{1}{2}$
   $x = 3\frac{1}{2}$
   Check: $3\frac{1}{2} + 3\frac{1}{2} = 7$
   $7=7$

   d) $3.7 = a + 11.2$
   $3.7 - 11.2 = a + 11.2 - 11.2$
   $-7.5 = a$
   Check: $3.7 = -7.5 + 11.2$
   $3.7 = 3.7$

3. a) 1) 1/3

   2) -2

   3) 1/a

   4) 3/2

   b) $13t = -52$ Check: $13(-4) = -52$
   $\frac{13t}{13} = \frac{-52}{13}$ $-52 = -52$
   $t = -4$

   c) $-8 = \frac{-1b}{5}$ Check:
   $(-5)(-8) = (-5)\left(\frac{-1b}{5}\right)$ $-8 = \frac{-1b}{5}$
   $40 = b$ $-8 = \frac{-1}{5}(40)$
   $-8 = -8$

   d) $-14k = 84$ Check: $-14(-6) = 84$
   $\frac{-14k}{-14} = \frac{84}{-14}$ $84 = 84$
   $k = -6$

4. a) A. 3

   B. 1

   C. 4

   D. 2

Answers (continued)

4. b) D

c) $5x - 9 = -19$

$5x - 9 + 9 = -19 + 9$

$5x = -10$

$\frac{5x}{5} = \frac{-10}{5}$

$x = -2$

Check: $5(-2) - 9 = -19$
$-10 - 9 = -19$
$-19 = -19$

d) $8 = 5x + 33$

$8 - 33 = 5x + 33 - 33$

$-25 = 5x$

$-\frac{25}{5} = \frac{5x}{5}$

$-5 = x$

Check: $8 = 5(-5) + 33$
$8 = -25 + 33$
$8 = 8$

5. a) 1) $-4x$

2) $2x$

3) $\frac{1}{2}x$

4) $-x$

b) $7x + 5 = 2x + 35$

$7x + 5 - 2x = 2x + 35 - 2x$

$5x + 5 = 35$

$5x + 5 - 5 = 35 - 5$

$5x = 30$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

5. b) (continued)

Check: $7(6) + 5 = 2(6) + 35$
$42 + 5 = 12 + 35$
$47 = 47$

c) $7y - 5 = 9y + 29$

$7y - 7y - 5 = 9y + 29 - 7y$

$-5 = 2y + 29$

$-5 - 29 = 2y + 29 - 29$

$-34 = 2y$

$-\frac{34}{2} = \frac{2y}{2}$

$-17 = y$

Check: $7(-17) - 5 = 9(-17) + 29$
$-119 - 5 = -153 + 29$
$-124 = -124$

d) $16 + 4y = 10y - 20$

$16 + 4y - 4y = 10y - 20 - 4y$

$16 = 6y - 20$

$16 + 20 = 6y - 20 + 20$

$36 = 6y$

$\frac{36}{6} = \frac{6y}{6}$

$6 = y$

Check: $16 + 4(6) = 10(6) - 20$
$16 + 24 = 60 - 20$
$40 = 40$

Answers (continued)

6. a)

$$5(3y - 2) = 5$$

$$15y - 10 = 5$$

$$15y - 10 + 10 = 5 + 10$$

$$15y = 15$$

$$\frac{15y}{15} = \frac{15}{15}$$

$$y = 1$$

Check:

$$\begin{aligned} 5(3 \cdot 1 - 2) &= 5 \\ 5(1) &= 5 \\ 5 &= 5 \end{aligned}$$

b)

$$7(x + 2) = 5(x + 4)$$

$$7x + 14 = 5x + 20$$

$$7x + 14 - 5x = 5x + 20 - 5x$$

$$2x + 14 = 20$$

$$2x + 14 - 14 = 20 - 14$$

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

Check:

$$\begin{aligned} 7(3 + 2) &= 5(3 + 4) \\ 7(5) &= 5(7) \\ 35 &= 35 \end{aligned}$$

c)

$$3d + 2(6d - 5) = 5$$

$$3d + 12d - 10 = 5$$

$$15d - 10 = 5$$

$$15d - 10 + 10 = 5 + 10$$

$$15d = 15$$

$$\frac{15d}{15} = \frac{15}{15}$$

$$d = 1$$

6. c) (continued)

Check:

$$\begin{aligned} 3(1) + 2(6 \cdot 1 - 5) &= 5 \\ 3 + 2(1) &= 5 \\ 3 + 2 &= 5 \\ 5 &= 5 \end{aligned}$$

d)

$$3(x + 2) + 3x + 2 = 74$$

$$3x + 6 + 3x + 2 = 74$$

$$6x + 8 = 74$$

$$6x + 8 - 8 = 74 - 8$$

$$6x = 66$$

$$\frac{6x}{6} = \frac{66}{6}$$

$$x = 11$$

Check:

$$\begin{aligned} 3(11 + 2) + 3(11) + 2 &= 74 \\ 39 + 33 + 2 &= 74 \\ 74 &= 74 \end{aligned}$$

7. a)
   1. Not Equivalent
   2. Equivalent
   3. Not Equivalent
   4. Equivalent

b)
   1. B
   2. D
   3. C
   4. A

c) Multiplying both sides of an equation by a negative constant has no effect on the equation. Multiplying both sides of an inequality by a negative constant reverses the inequality.

Answers (continued)

7. d) 1. Subtraction Property of Inequality

   2. Substitution

   3. Division Property of Inequality

   4. Substitution

8. a) 1. $>$

   2. $<$

   3. $<$

   4. $<$

b)

$$b + 60 < -100$$
$$b + 60 - 60 < -100 - 60$$
$$b < -160$$

c)

$$-12x + 1 \geq 25$$
$$-12x + 1 - 1 \geq 25 - 1$$
$$-12x \geq 24$$
$$\frac{-12x}{-12} \leq \frac{24}{-12}$$
$$x \leq -2$$

d)

$$16 - 4n \leq 6n - 24$$
$$16 - 4n + 4n \leq 6n - 24 + 4n$$
$$16 \leq 10n - 24$$
$$16 + 24 \leq 10n - 24 + 24$$
$$40 \leq 10n$$
$$\frac{40}{10} \leq \frac{10n}{10}$$
$$4 \leq n$$

Answers (continued)

9. a) $x + 6 > 8$ and $x - 1 < 4$

$x + 6 - 6 > 8 - 6$ $\quad x - 1 + 1 < 4 + 1$

$x > 2$ and $x < 5$

b) $x - 3 \leq -4$ or $x - 3 \geq 4$

$x - 3 + 3 \leq -4 + 3$ $\quad x - 3 + 3 \geq 4 + 3$

$x \leq -1$ or $x \geq 7$

c) $-2 \geq x + 6 \leq 3$

$-2 \leq x + 6$ and $x + 6 \leq 3$

$-2 - 6 \leq x + 6 - 6$ $\quad x + 6 - 6 \leq 3 - 6$

$-8 \leq x$ and $x \leq -3$

$-8 \leq x \leq 3$

d) $y + 6 > 7$ or $3y + 5 < 3$

$y + 6 - 6 > 7 - 6$ $\quad 3y + 5 - 5 > 3 - 5$

$y > 1$ $\quad 3y < -2$

$\frac{3y}{3} < -\frac{2}{3}$

$y < -\frac{2}{3}$

10. a) C

b) D

c) $6x + 2 - 8x < 14$

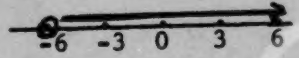
$-2x + 2 < 14$

$-2x + 2 - 2 < 14 - 2$

$-2x < 12$

$-\frac{24}{-2} > \frac{12}{-2}$

$x > -6$



d) $5x \leq 10 + 2(3x - 4)$

$5x \leq 10 + 6x - 8$

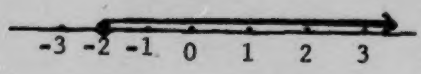
$5x \leq 2 + 6x$

$5x - 6x \leq 2 + 6x - 6x$

$-x \leq 2$

$\frac{-x}{-1} \geq \frac{2}{-1}$

$x \geq -2$



Answers (continued)

11. a) $|2x + 3| = 11$

$2x + 3 = 11$ or $2x + 3 = -11$

$2x = 8$ $\quad$ $2x = -14$

$x = 4$ or $x = -7$

b) $|x + 7| - 3 \geq 9$

$|x + 7| \geq 12$

$x + 7 \geq 12$ or $x + 7 \leq -12$

$x \geq 5$ or $x \leq -19$

c) $5 + |2x - 5| \leq 7$

$|2x - 5| \leq 2$

$-2 \leq 2x - 5$ and $2x - 5 \leq 2$

$3 \leq 2x$ and $2x \leq 7$

$1\frac{1}{2} \leq x$ and $x \leq 3\frac{1}{2}$

$1\frac{1}{2} \leq x \leq 3\frac{1}{2}$

d) $|3x + 4| < 13$

$-13 < 3x + 4$ and $3x + 4 < 13$

$-17 < 3x$ and $3x < 9$

$-5\frac{2}{3} < x$ and $x < 3$

$-5\frac{2}{3} < x < 3$

12. a) 1. C

2. D

3. A

4. B

b) $5x = 6 + 2x$

c) $7x < 2(x + 7)$

d) $2(x + 7) = 2x + 14$

13. a) 1. Let x = number of hours

2. $d = 30x$

3. $d = 40x$

b) 1. Let x = 1st integer

2. Let x + 1 = 2nd integer

c) 1. Mr. Roberts = x + 2

2. Mr. Robert's distance

$= 45(x + 2)$

3. Tony's distance = 55x

d) 1. x + 2 = 2nd even integer

2. x + 4 = 3rd even integer

Answers (continued)

14. a) Let x = 1st integer

$x + 1$ = 2nd integer

$\therefore x + (x + 1) = 57$

b) Let w = number of hours

$30(w)$ = slow car's distance

$40(w)$ = fast car's distance

$\therefore 30w + 40w = 2100$

c) Let x = pounds of \$2.50 coffee

$70 - x$ = pounds of \$2.87 coffee

$\therefore \$2.50(x) + \$2.87(70-x)$

$= \$2.61(70)$

d) Let x = 1st integer

$x + 2$ = 2nd integer

$\therefore x + (x + 2) < 80$

15. a) Let x = Elaine's books

$2x + 1$ = Barry's books

$\rightarrow 2x + 1 = 59$

$2x = 58$

$x = 29$

Elaine has 29 books

15. b) Let x = games lost

$2x$ = games won

$\rightarrow 2x = 18$

$x = 9$

Lost 9 games

c) Let x = number of 25¢ notebooks

$348 - x$ = number of 38¢ notebooks

$\rightarrow .25x + .38(348 - x) = 100.91$

$.25x + 132.24 - .38x = 100.91$

$-.13x + 132.24 = 100.91$

$-.13x = -31.33$

$x = 241$

$348 - x = 107$

d) Let x = number of hours

$350x$ = distance west

$400x$ = distance east

$\rightarrow 350x + 400x = 1500$

$750x = 1500$

$x = 2$

Time: 10:00 a.m.

UNIT V - GRAPHING

PURPOSE

Graphing forms a bridge between the study of algebra and geometry. This section of the course introduces the students to Cartesian coordinates and their application to linear open sentences. The knowledge of linear functions and their characteristics provides students with alternative methods for solving open sentences.

OVERVIEW

Students become acquainted with the basic terminology of rectangular coordinates and the location of points in the system. Graphing of linear equations is developed so that a functional level of understanding of equations and their graphs is obtained. The relationships between a line and the equation which defines it are discussed. The coordinate system is also used to graph linear inequalities and higher order functions.

SUGGESTIONS TO THE TEACHER

Instructional Days: 9-12
Minimal Course Objectives: #1-10, 14
Average Course Objectives: ALL
Maximal Course Objectives: ALL

This unit is an excellent one for incorporating multimedia techniques such as the overhead projector and colored chalk to stimulate student interest.

There are numerous supplementary resources available which enable students to obtain identifiable pictures as a result of graphing ordered pairs. Advanced students may enjoy drawing original pictures on graph paper (using only straight lines) and then determining the equation for each line.

When discussing the slope of a line, it is helpful to speak constantly of the line going from left to right.

Games such as "Battleship" can be readily adapted for practice in coordinate graphing.

VOCABULARY

abscissa (x-coordinate)
Cartesian coordinates
coordinates
domain
function
half plane
ordered pair
ordinate (y-coordinate)
origin
plane
quadrant
range
relation
rectangular coordinates
slope
x-axis (horizontal axis)
y-axis (vertical axis)
y-intercept

UNIT V - GRAPHING

ENTERING PERFORMANCE OBJECTIVES

1. Add, subtract, multiply, and divide directed numbers.
2. Graph an ordered pair of numbers on the coordinate plane.
3. Evaluate algebraic expressions.
4. Solve equations for a given variable.

Assessment Tasks

1. a) $-7 + (-12)$ a) ____________

b) $-2\frac{2}{3} + (-1\frac{1}{3})$ b) ____________

c) $-5\frac{1}{4} + 2\frac{3}{8}$ c) ____________

d) $-6.4 + 16.6 + (-12.9) + 8.4$ d) ____________

e) $3\frac{1}{3} + (-11\frac{2}{3})$ e) ____________

f) $-5.8 - 8.8$ f) ____________

g) $8.4 - 12.9$ g) ____________

h) $1\frac{7}{8} - 7\frac{3}{8}$ h) ____________

i) $-12\frac{1}{3} - (-3\frac{1}{2})$ i) ____________

j) $-\frac{7}{12} - \frac{2}{5}$ j) ____________

k) $(5)(-7)(-3)$ k) ____________

l) $(-0.75)(-3)$ l) ____________

m) $(-2\frac{5}{8})(4\frac{2}{3})$ m) ____________

n) $(-\frac{1}{8})(-\frac{1}{3})$ n) ____________

o) $(2.5608) \div (-0.8)$ o) ____________

p) $(-75) \div 15$ p) ____________

q) $(-3\frac{4}{5}) \div (-\frac{1}{3})$ q) ____________

r) $(-144) \div (-9)$ r) ____________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. Graph the following ordered pairs of numbers, and label each point with its coordinates.

   a) (5,0)
   b) (0,7)
   c) (-3,0)
   d) (0,-4)
   e) (3,4)
   f) (-2,6)
   g) (-3,-8)
   h) (3,-5)
   i) (7,3)
   j) (-8,2)
   k) (-5,-7)
   l) (7,-4)

3. If $x = 2$, $y = -3$, and $z = -1$, find the value of each of the following:

   a) $xy + xz - yz$ a) ____________
   b) $x^3y - x^2z + x + 20$ b) ____________
   c) $x^2yz - 2z$ c) ____________
   d) $x^3y^2 + x^2z + xy - 4$ d) ____________

4. Solve each equation for x, y, or z.

   a) $2x + 5w = 1$ a) ____________
   b) $P = 2x + 2w$ b) ____________
   c) $s + ry = t$ c) ____________
   d) $4v - 3y = 6$ d) ____________

UNIT V - GRAPHING

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) -19

b) -4

c) $-2\frac{7}{8}$

d) 5.7

e) $-8\frac{1}{3}$

f) -14.6

g) -4.5

h) $-5\frac{1}{2}$

i) $-8\frac{5}{6}$

j) $-\frac{59}{60}$

k) 105

l) 2.25

m) $-\frac{49}{4}$ or $-12\frac{1}{4}$

n) $\frac{1}{24}$

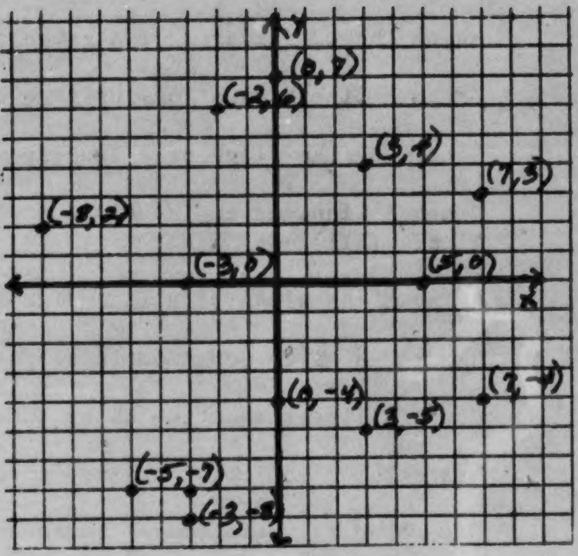
o) -3.201

p) -5

q) $11\frac{2}{5}$

r) 16

2.



3. a) -11

b) 2

c) 14

d) 58

4. a) $\frac{-5w + 1}{2} = x$

b) $x = \frac{P - 2w}{2}$

c) $y = \frac{t - s}{r}$

d) $y = \frac{4v}{3} - 2$

UNIT V - GRAPHING

PERFORMANCE OBJECTIVES

1. Distinguish among axes, origin, quadrants, and ordered pairs when given an illustration of a rectangular coordinate system. (I)
2. Graph ordered pairs on a coordinate plane. (II)
3. State the coordinates of a given point. (II)
4. Graph a linear equation, using a table of values. (III)
5. Determine the slope of a line from its graph. (II)
6. Graph a line, given its slope and the coordinates of a point on the line. (III)
7. Compute the slope of a line algebraically. (II)
8. Rewrite a given equation in slope-intercept form. (II)
9. Graph a linear equation using the slope and y-intercept. (III)
10. Write the equation of a line, given its slope and y-intercept. (II)
11. Determine the equation of a line, given its slope and the coordinates of a point on the line. (III)
12. Determine the equation of a line, given the coordinates of two points on the line. (III)
13. Determine whether a given relation is a function. (II)
14. Graph a linear inequality. (III)
15. Graph a quadratic equation, using a table of values. (III)

Minimal	Average	Maximal
#1 - 10, 14	All	All

KEY SKILLS FOR END-OF-COURSE TESTING

10. Graph linear equations in the coordinate plane.
11. Graph linear inequalities in the coordinate plane.
12. Determine an equation of a line, given the slope and a point, or two points of the line.

UNIT V - GRAPHING

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	296-298	202	296-297	321	208-210	120-121	169-176	264
2	296-298	203	296-299	322-324	210-211	121-122	178	265-268
3	296-298	204	296-299	324	210-211	122	175	266
4	311-313	208-212	310-313	326-329 340-342	215-216	126-128	181-182	269-271
5	315-317	231	396-399	349-353	226	139-142	213-214	275-276
6	316-317	237	400	---	231	141	---	279-280 287
7	315-316	236	396-399	357	230	149	215	277
8	320-322	233	406-408	382-384	235	147	220-222	282-284
9	320-322	234	402-405	348, 363	235-236	149	220-222	282-284
10	324-325	232-233	414-416	348-349	---	149	227-230	285-286
11	324-325	234-235	414-416	---	---	149	227-230	279-280 287
12	324-325	---	415	---	---	151-153	227-230	266-287
13	301-303	219-221	314-318	330-336	499	134-138	195-197	264-269
14	360-362	240-243	324-326	---	286	162-165	184-188	297-299
15	416	439-443	370-374	344-347	216	129	435	

UNIT V - GRAPHING

CROSS REFERENCES

TEXTS (BY AUTHOR)

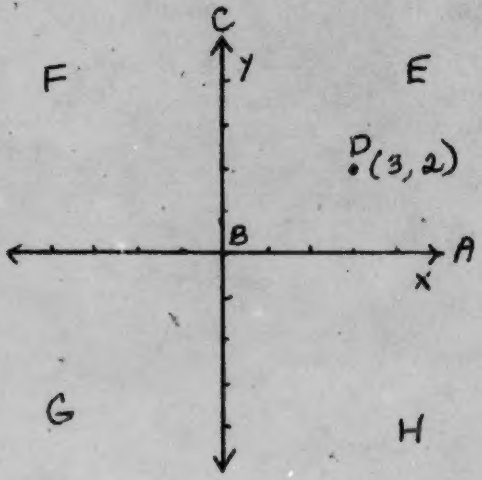
Objectives	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
1	117 255	377-378	102,105	261-262	59
2	255	378,381	106,107	263-264	60
3	255	378	104	265-266	60
4	255 258-259	387,392	108-110	271-274	73
5	261	---	251	285-289	73
6	---	---	252	---	---
7	265	---	249-252	289	63
8	268	---	246-248	294	65
9	271	---	252	292-294	329
10	---	---	252	298	---
11	---	---	252	299	---
12	---	---	250-252	300	---
13	---	383-384	111-114	281, 282	---
14	288-294	---	381-387	234	---
15	296-299	387	401-403	---	389-392

PERFORMANCE OBJECTIVE V-1

Distinguish among axes, origin, quadrants, and ordered pairs when given an illustration of a rectangular coordinate system.

a) Indicate, by the correct letter(s), illustrations of each of the following in the diagram at the right.

1. axes ____________________
2. origin ____________________
3. quadrants ________________
4. ordered pairs ____________



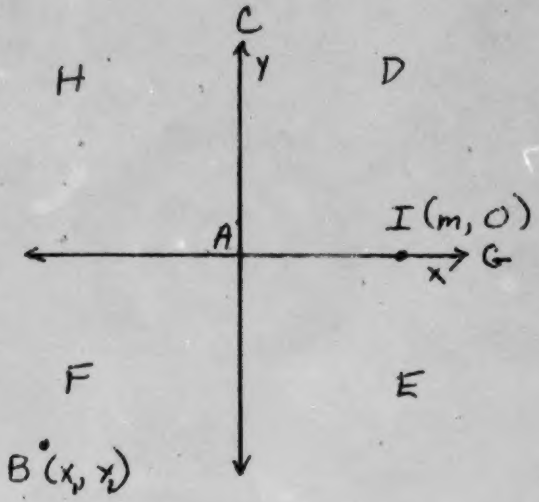
b) Match each name to its correct illustrations by writing the letter of the correct answer on the blank.

___ 1. axes　　A. D,E,F,H

___ 2. origin　　B. B,I

___ 3. quadrants　　C. C,G

___ 4. ordered pairs　　D. A



PERFORMANCE OBJECTIVE V-1

Distinguish among axes, origin, quadrants, and ordered pairs when given an illustration of a rectangular coordinate system.

c) In the drawing at the right, R and B represent which of the following?

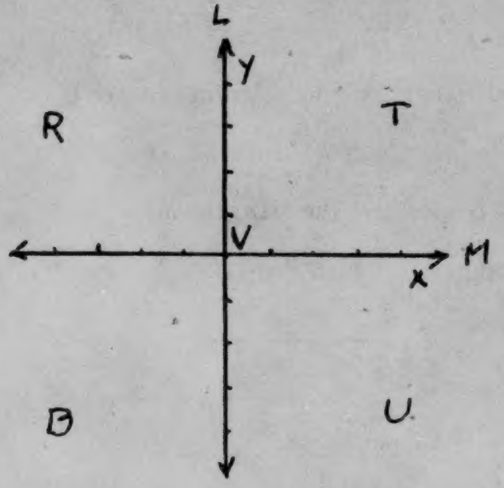
A. axes

B. origin

C. quadrants

D. ordered pairs

Answer ______________________



d) In the drawing at the right, C, F, and X represent which of the following?

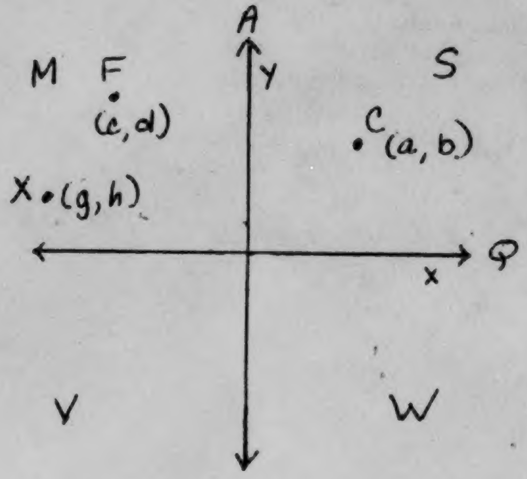
A. axes

B. origin

C. quadrants

D. ordered pairs

Answer ______________________

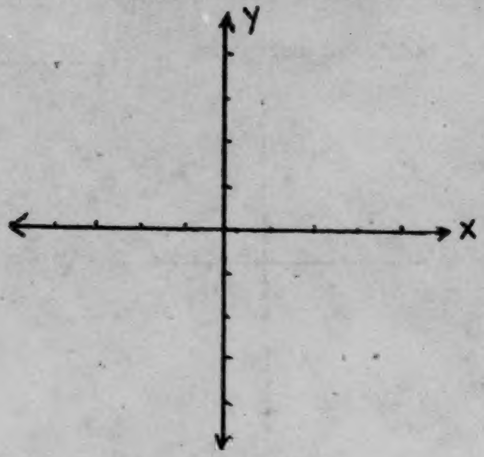


PERFORMANCE OBJECTIVE V-2

Graph ordered pairs on a coordinate plane.

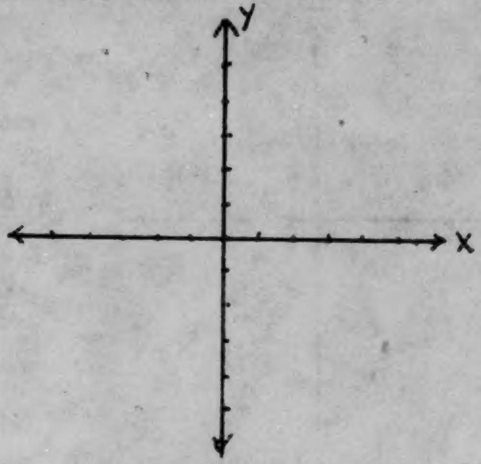
a) On the graph below, plot each of the following points; and label with the appropriate letter.

A. (-1,3)
B. (0,0)
C. (2,4)
D. (3,-1)



b) On the graph below, plot each of the following points; and label with the appropriate letter.

A. (5,0)
B. (0,-3)
C. (2,5)
D. (-4,-1)

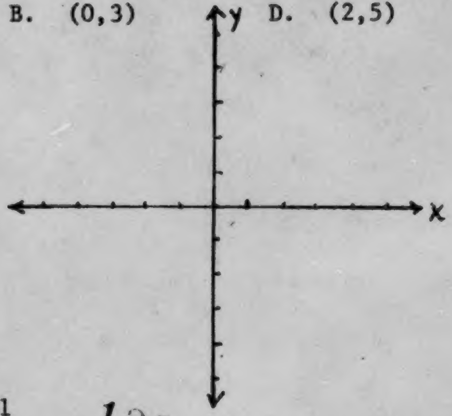


c) Identify the quadrant that contains each of the following points.

Point	Quadrant
1. (6,3)	________
2. (-2,4)	________
3. (3,-1)	________
4. (-2,-2)	________

d) On the graph below, plot the following vertices of a triangle. Label each vertex and draw the sides.

A. (-4,0)
B. (0,3)
C. (2,-2)
D. (2,5)

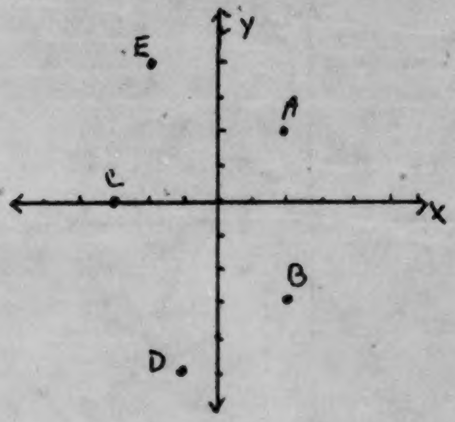


PERFORMANCE OBJECTIVE V-3

State the coordinates of a given point.

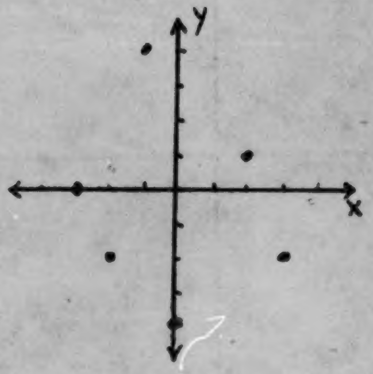
a) State the coordinate of each point graphed below.

1. point A = ___
2. point B = ___
3. point C = ___
4. point D = ___
5. point E = ___



b) From the diagram, name the coordinates of each point described below.

1. point in Quadrant I __________
2. point in Quadrant IV __________
3. point in Quadrant III __________
4. point on the y-axis __________



c) Name the coordinates of each point described below.

_____ 1. the point located 4 units up from the origin

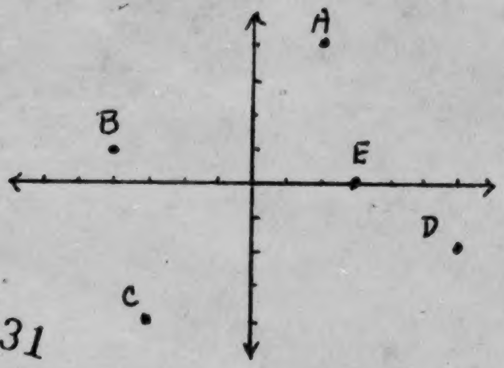
_____ 2. the point located on the x-axis 4 units to the left of the origin

_____ 3. the point 2 units to the right of (3,-1)

_____ 4. the point 3 units below (2,-3)

d) State the coordinate of each point shown on the graph below.

1. point A _____
2. point B _____
3. point C _____
4. point D _____
5. point E _____



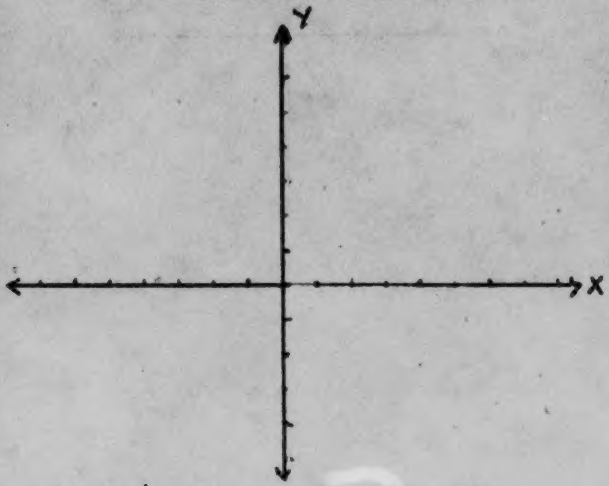
PERFORMANCE OBJECTIVE V-4

Graph a linear equation, using a table of values.

Complete the following table and then use the values to draw the graph.

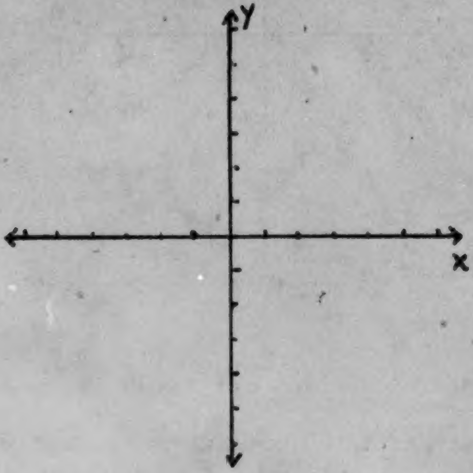
a) $x + y = 6$

x	2	3		-1	
y			0		-2



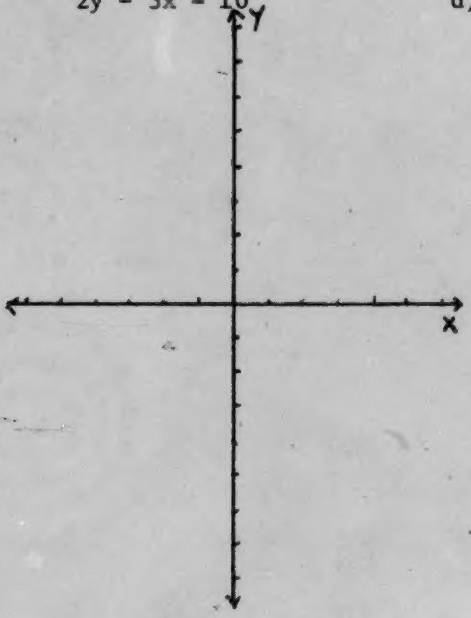
b) $2x = y$

x	0			-1	-2
y		2	6		



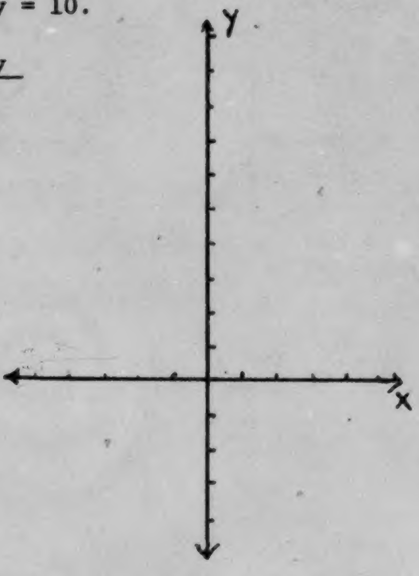
c) $2y - 3x = 10$

x	y
-4	—
-2	—
0	—
—	—
—	—



d) Make a table of three pairs of values for x and y that satisfy the equation $2x + y = 10$.

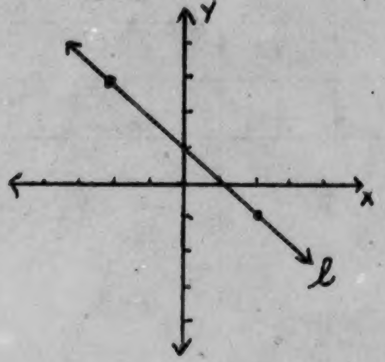
x	y



PERFORMANCE OBJECTIVE V-5

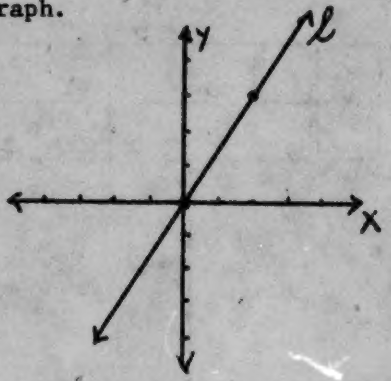
Determine the slope of a line from its graph.

a) Determine the slope of the line from its graph.



Answer ______________________

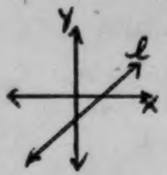
b) Determine the slope of the line from its graph.



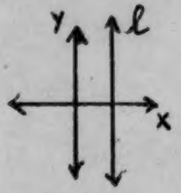
Answer ______________________

c) Determine which diagram below satisfies the conditions described.

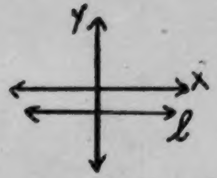
A



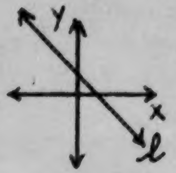
B



C

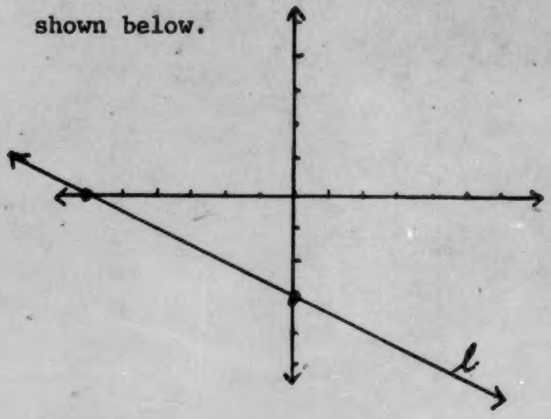


D



1. a positive slope __________
2. a negative slope __________
3. slope of zero __________
4. no slope __________

d) Determine the slope of the line shown below.

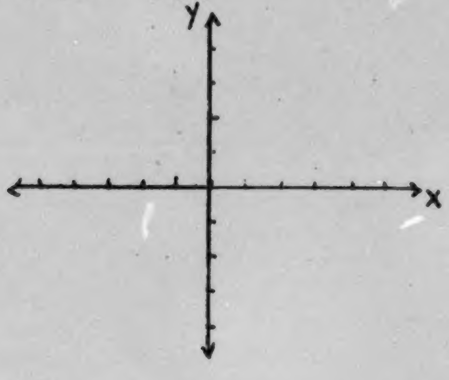


Answer ______________________

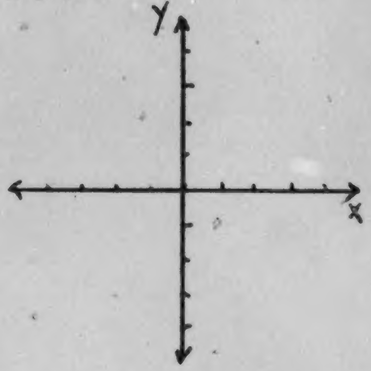
PERFORMANCE OBJECTIVE V-6

Graph a line, given its slope and the coordinates of a point on the line.

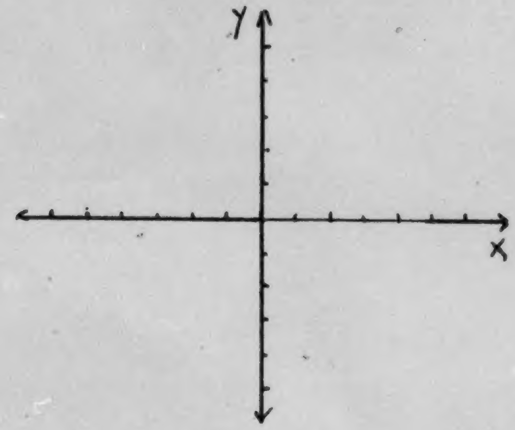
a) Draw the graph of a line that has a slope of 3 and passes through the point (0,2).



b) Draw the graph of a line that has a slope of $^{-}\frac{2}{3}$ and passes through the point (0,0).



c) Draw a line which passes through the point (5,-3) and has no slope.



d) Determine two other points on the line which passes through (-2,-3) and has a slope of 3.

Answers __________ __________

PERFORMANCE OBJECTIVE V-7

Compute the slope of a line algebraically.

a) Find the slope of a line passing through the points (4,0) and (2,-1).

Answer ______________________

b) What is the slope of a line passing through the points (a,0) and (0, a) ?

Answer ______________________

c) The slope of a line is -2. It passes through the points (1,4) and (5,?). Find the missing coordinate.

Answer ______________________

d) Which of the following is a formula for determining the slope of a line?

A. $M = \frac{x_2 - x_1}{y_2 - y_1}$

B. $M = \frac{y}{x}$

C. $M = \frac{y_2 - y_1}{x_2 - x_1}$

D. None of the above.

Answer ______________________

PERFORMANCE OBJECTIVE V-8

Rewrite a given equation in slope-intercept form.

a) Which of the following is the correct slope-intercept form of the equation $3x + 2y = 3$?

A. $y = 3x + 3$

B. $y = \frac{3}{2}x + 1$

C. $y = {}^{-}\frac{3}{2}x - 1$

D. = none of the above

Answer ____________________

b) Rewrite the following equation in slope-intercept form: $x + 2y = 6$.

Answer ____________________

c) What is the slope of the line defined by the equation $2x - y = 3$?

Answer ____________________

d) Solve the following equation for y: $3x + 4y = 8$.

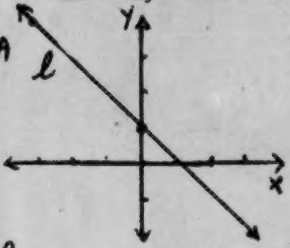
Answer ____________________

PERFORMANCE OBJECTIVE V-9

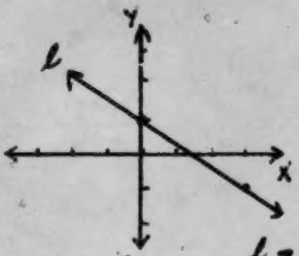
Graph a linear equation, using the slope and y-intercept.

a) Which of the following is the graph of the line for the equation $2x - 3y = 3$?

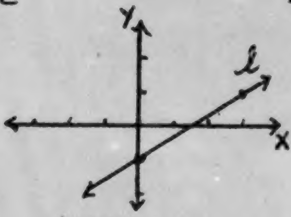
A



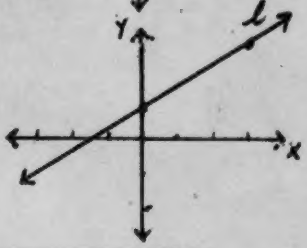
B



C

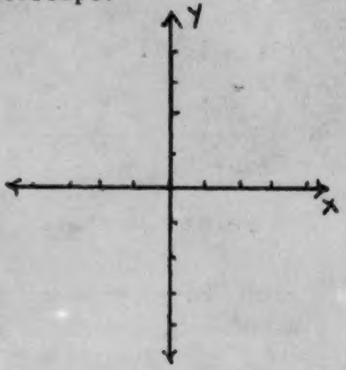


D

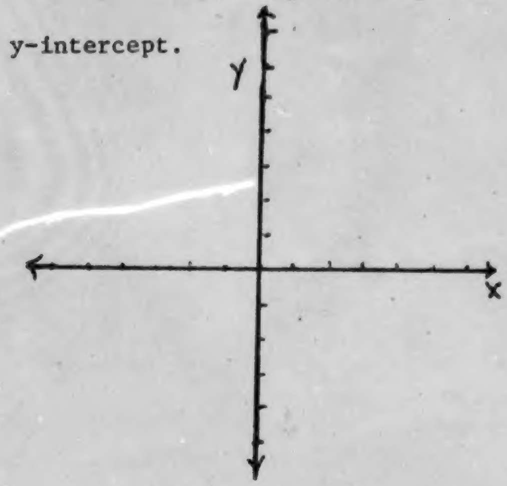


Answer ________________________

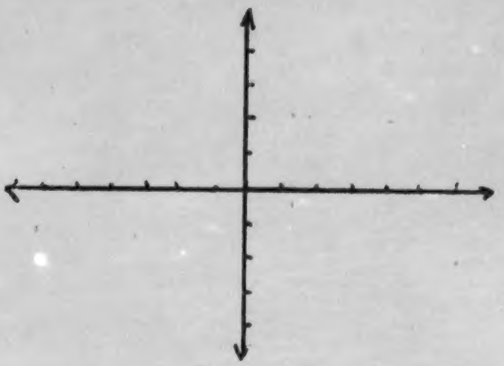
b) Graph the line of the equation $3x + y = 0$ by using the slope and y-intercept.



c) Draw the graph of the equation $-2x + y = 6$ by using the slope and y-intercept.



d) Graph the following equation the slope and y-intercept: $5y + 10 = -2x$.



PERFORMANCE OBJECTIVE V-10

Write the equation of a line, given its slope and y-intercept.

a) A line has a slope of 3 and a y-intercept of $\frac{1}{2}$. The equation that describes the relationship between x and y coordinates of the graph is:

A. $\frac{1}{2}x + 3 = y$

B. $3x + y = \frac{1}{2}$

C. $y = 3x + \frac{1}{2}$

D. $y = \frac{1}{2}x - 3$

Answer C

Write the equations for the graphs with the following characteristics.

b) y-intercept of (0,4) and slope of 2.

Answer ____________________

c) Slope of zero and y-intercept of (0,2).

Answer ____________________

d) Slope of $^{-}\frac{3}{2}$ and y-intercept at the origin.

Answer ____________________

PERFORMANCE OBJECTIVE V-11

Determine the equation of a line, given its slope and the coordinates of a point on the line.

a) Given a line passing through the point (3,0) and slope of $\frac{1}{2}$, write an equation for the line.

Answer ____________________

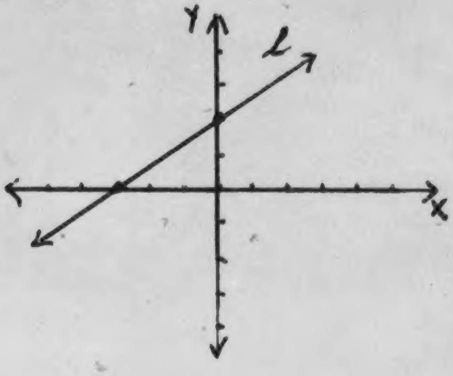
b) Given a line crossing the y-axis at -2 and a slope of 3, write an equation for the line.

Answer ____________________

c) Given a line passing through the origin and a slope of -3, write an equation for the line.

Answer ____________________

d) Determine the equation of the line graphed below by finding its slope and y-intercept.



Answer ____________________

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PERFORMANCE OBJECTIVE V-12

Determine the equation of a line, given the coordinates of two points on the line.

a) Write the equation of a line passing through the points (2,3) and (3,2).

Answer ________________________

b) A line passes through the origin and the point (-3,5). Find the equation of the line.

Answer ________________________

c) Find the equation of the line passing through the points (3,4) and (2,-5).

Answer ________________________

d) What is the equation of the line passing through the points (-6,3) and (3,-3)?

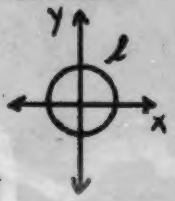
Answer ________________________

PERFORMANCE OBJECTIVE V-13

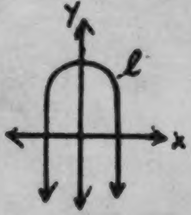
Determine whether a given relation is a function.

a) Which of the following illustrate a function?

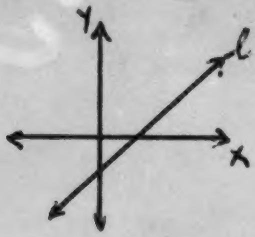
A



B



C



Answer ____________________

c) Which equations define a function?

A. $y = 3x - 4$

B. $x = -8$

C. $y = -9$

D. None of the above.

Answer ____________________

b) Which of the following sets of ordered pairs is a function?

A. {(2,3), (3,4), (4,5)}

B. {(3,0), (0,2), (3,-1), (2,0)}

C. {(-1,2), (1,2), (2,3), (2,-3)}

D. None of the above

Answer ____________________

d) Which of the following sets of ordered pairs represent functions?

A. {(1,6), (2,4), (3,2), (4,0), (6,-2)}

B. {(-1,-1), (0,0), (3,1), (0,-2), (3,-3)}

C. {(1,-4), (0,-3), (2,-3), (1,0), (3,0)}

D. {(0,0), (1,-6), (-1,4), (2,-8), (-2,0), (3,0), (-3,-18)}

E. All of the above

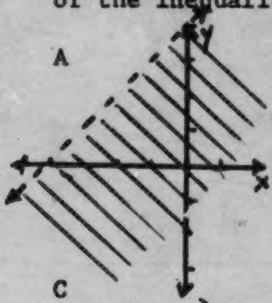
Answer ____________________

PERFORMANCE OBJECTIVE V-14

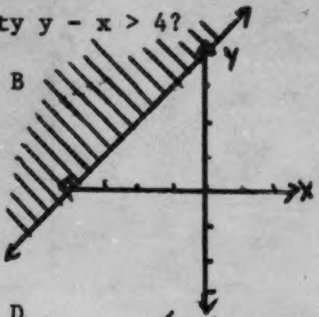
Graph a linear inequality.

a) Which of the following is the graph of the inequality $y - x > 4$?

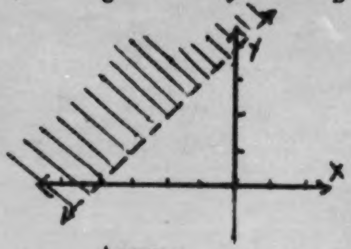
A



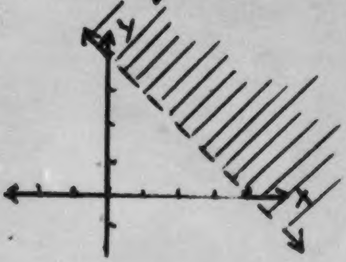
B



C

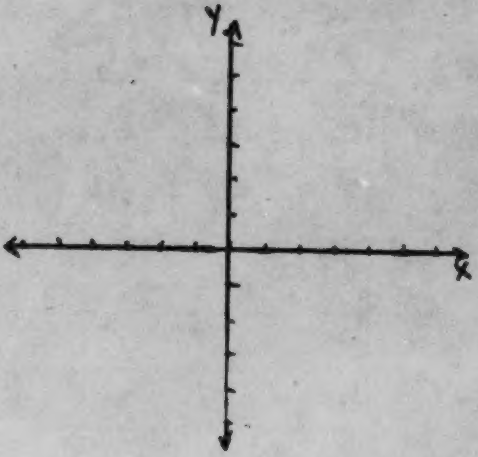


D

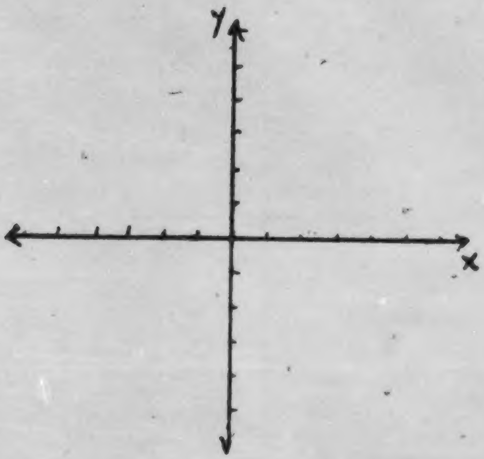


Answer ______________________

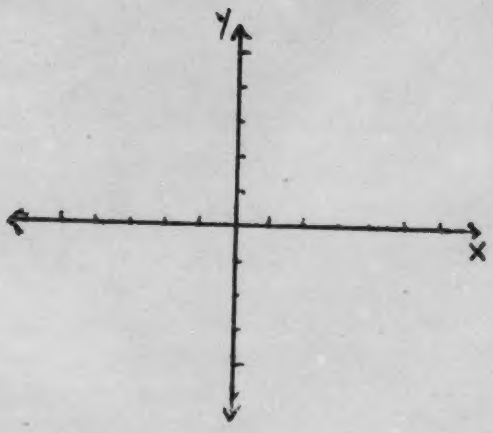
b) Graph the inequality $y > 3x - 4$.



c) Graph the inequality $2x + y \geq -3$.



d) Graph the inequality $y < -\frac{1}{3}x - 1$.

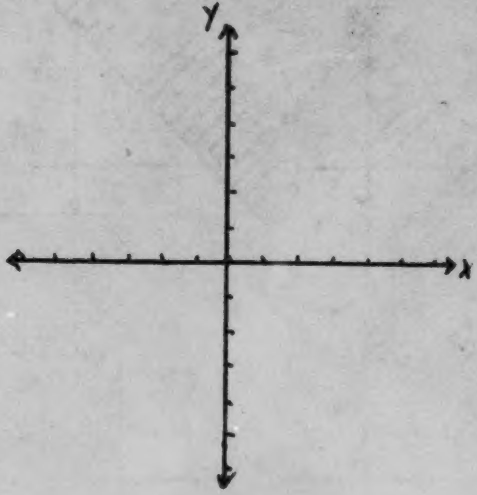


PERFORMANCE OBJECTIVE V-15

Graph a quadratic equation, using a table of values.

a) Find the appropriate points by completing the table, and use them to graph the equation $y = x^2 + 2x - 3$.

x	$x^2 + 2x - 3$	y
0		
1		
-3		
-1		
2		



b) Determine which equation defines the graph below.

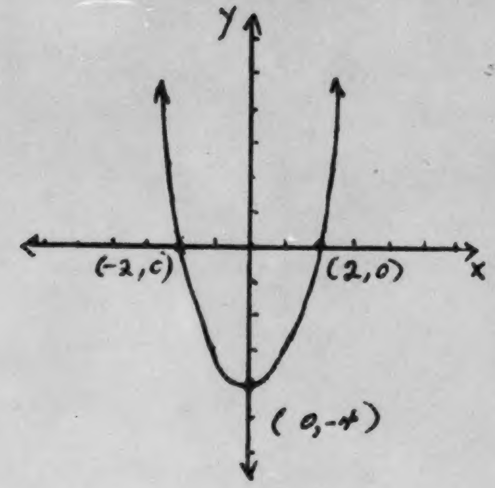
A. $y = x^2 - 4x + 4$

B. $y = x^2 - 4$

C. $y = x^2 + 4x + 4$

D. None of the above.

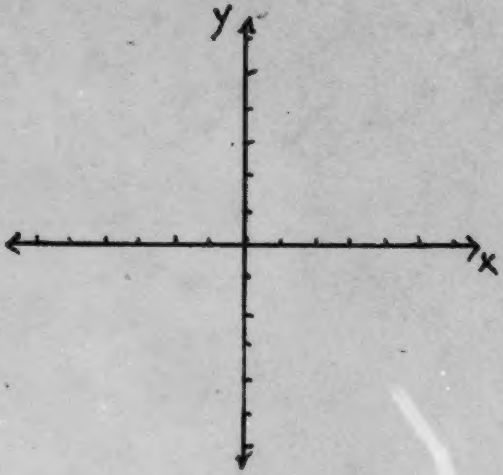
Answer ______________________



Graph a quadratic equation, using a table of values.

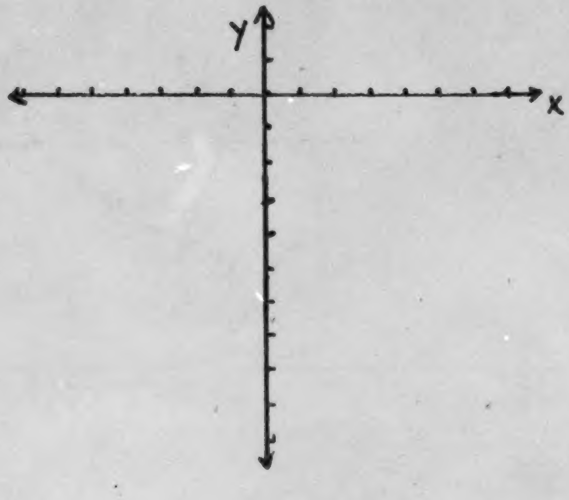
c) Graph the quadratic function $y = x^2 - 4$. Use the accompanying chart.

x	$x^2 - 4$	y
0		
1		
2		
-1		
-2		



d) Complete the following table of values, and graph the equation $y - 6x = -x^2 - 9$.

x		y
0		__
1		__
2		__
3		__
__		__
__		__
__		__



UNIT V - GRAPHING

Answers

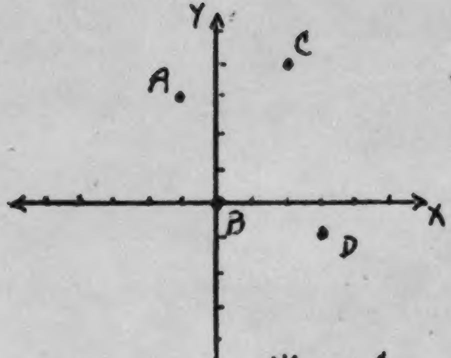
1. a) 1. A, C
2. B
3. E, F, G, H
4. D

b) 1. C
2. D
3. A
4. B

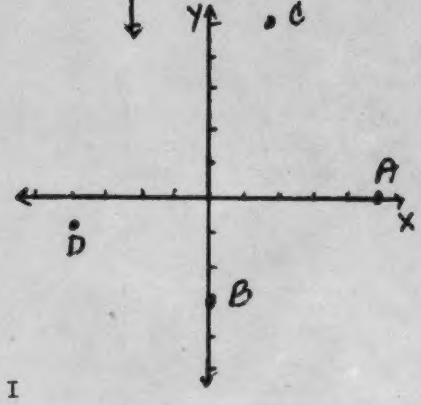
c) C

d) D

2. a)

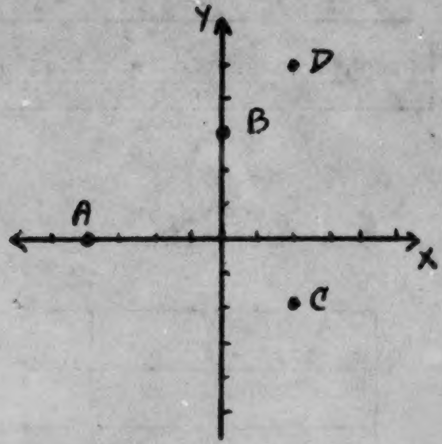


b)



c) 1. I
2. II
3. IV
4. III

d)



3. a) 1. (2,2)
2. (2,-3)
3. (-3,0)
4. (-1,-5)
5. (-2,4)

b) 1. (2,1)
2. (3,-2)
3. (-2,-2)
4. (0,-4)

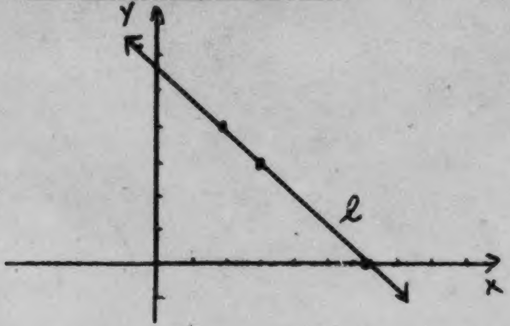
c) 1. (0,4)
2. (-4,0)
3. (5,-1)
4. (2,-6)

d) 1. (2,4)
2. (-4,1)
3. (-3,-4)
4. (6,-2)
5. (3,0)

Answers (continued)

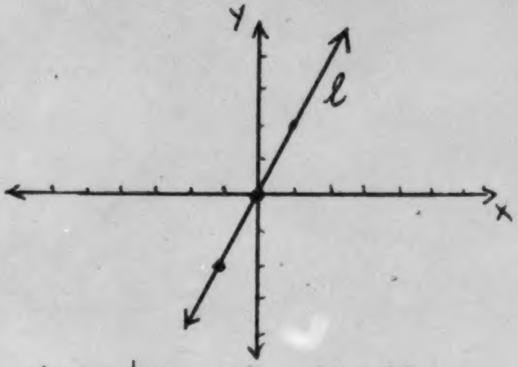
4. a)

x	2	3	6	-1	8
y	4	3	0	7	-2



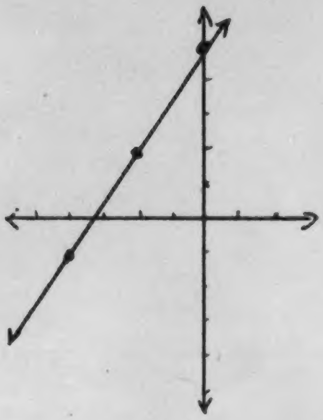
b)

x	0	1	3	-1	-2
y	0	2	6	-2	-4



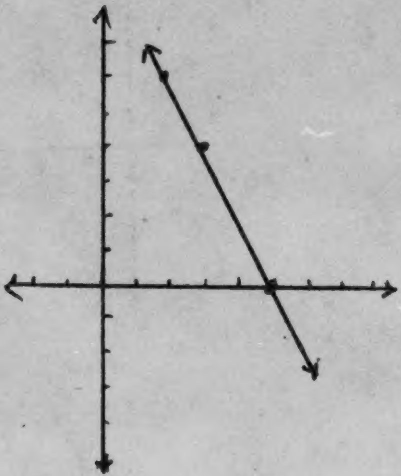
c) $2y - 3x = 10$

x	y
-4	-1
-2	2
0	5
-6	-4
2	8



d) $2x + y = 10$

x	y
3	4
2	6
0	10
5	0



5. a) -1

b) $\frac{3}{2}$

c) 1. A

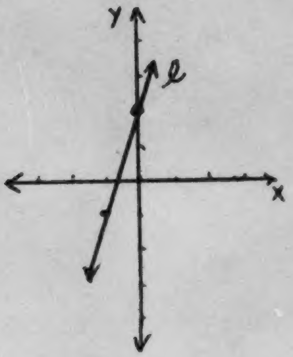
2. D

3. C

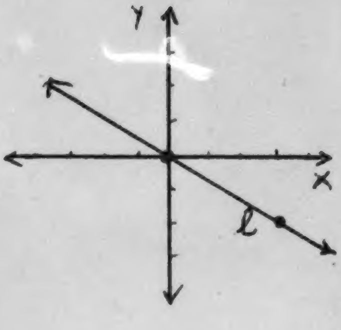
4. B

d) $-\frac{1}{2}$

6. a)

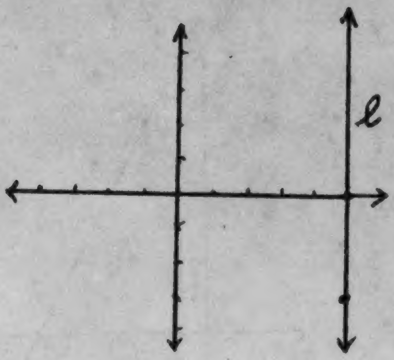


b)



Answers (continued)

6. c)



d) (-1,0)

(0,3)

7. a) $\frac{1}{2}$

b) -1

c) -4

d) C

8. a) D

b) $y = {}^{-}\frac{1}{2}x + 3$

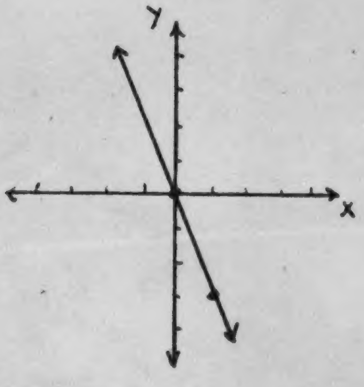
c) 2

d) $y = {}^{-}\frac{3}{4}x + 2$

9. a) C

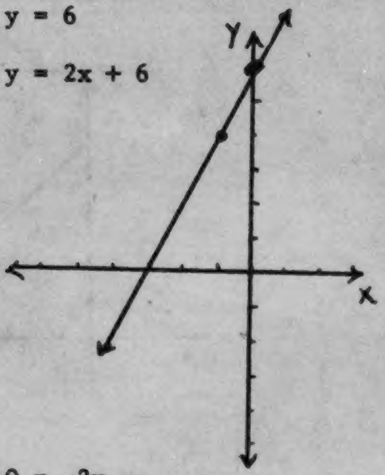
b) $3x + y = 0$

$y = -3x$



c) $-2x + y = 6$

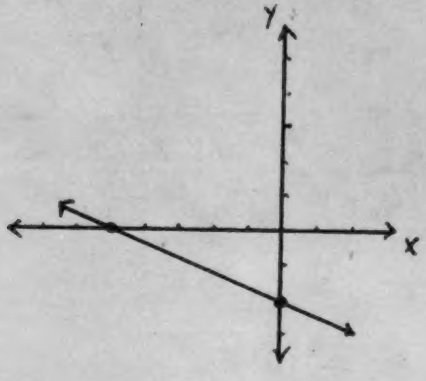
$y = 2x + 6$



d) $5y + 10 = -2x$

$5y = -2x - 10$

$y = {}^{-}\frac{2}{5}x - 2$



10. a) C

b) $y = 2x + 4$

c) $y = 2$

d) $y = {}^{-}\frac{3}{2}x$

11. a) $y = \frac{1}{2}x - \frac{3}{2}$

b) $y = 3x - 2$

c) $y = -3x$

d) $y = \frac{2}{3}x + 2$

Answers (continued)

12. a) $y = -x + 5$

    b) $y = {}^{-}\frac{5}{3}x$

    c) $y = 9x - 23$

    d) $y = {}^{-}\frac{2}{3}x - 1$

13. a) B, C

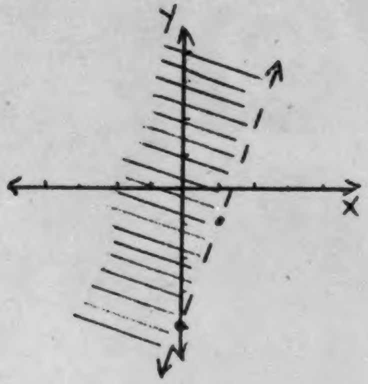
    b) A

    c) A, C

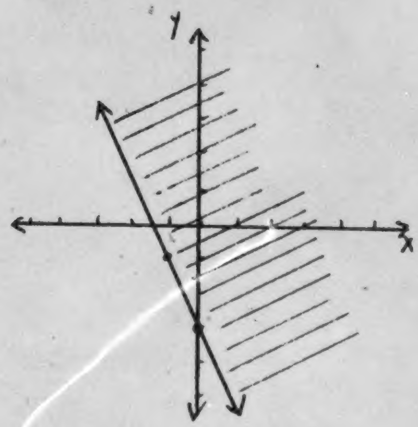
    d) A, D

14. a) C

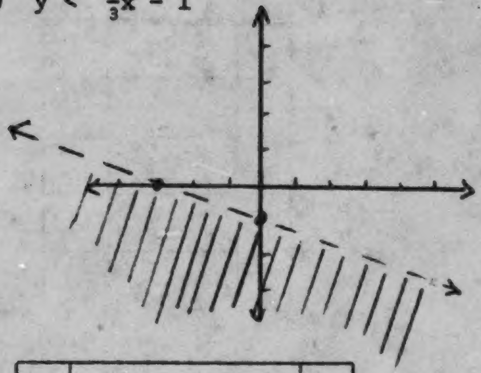
    b) $y > 3x - 4$



c) $2x + y \geq -3$

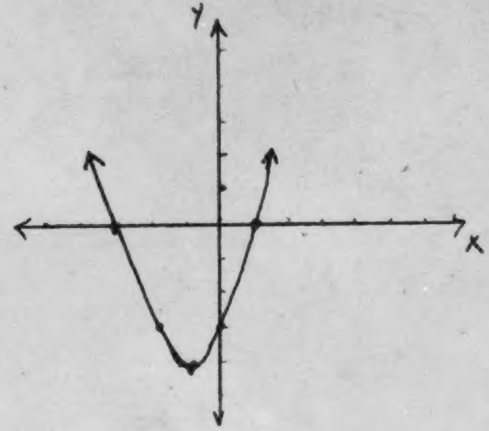


d) $y < {}^{-}\frac{1}{3}x - 1$



15 a)

x	$x^2 + 2x + -3$	y
0	$0^2 + 2(0) - 3$	-3
1	$1^2 + 2(1) - 3$	0
-3	$-3^2 + 2(-3) - 3$	0
-1	$-1^2 + 2(-1) - 3$	-4
2	$2^2 + 2(2) + -3$	5

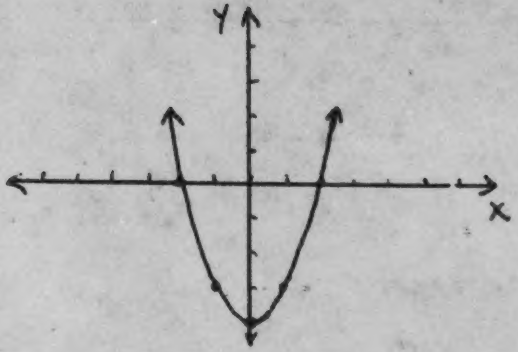


b) B

Answers (continued)

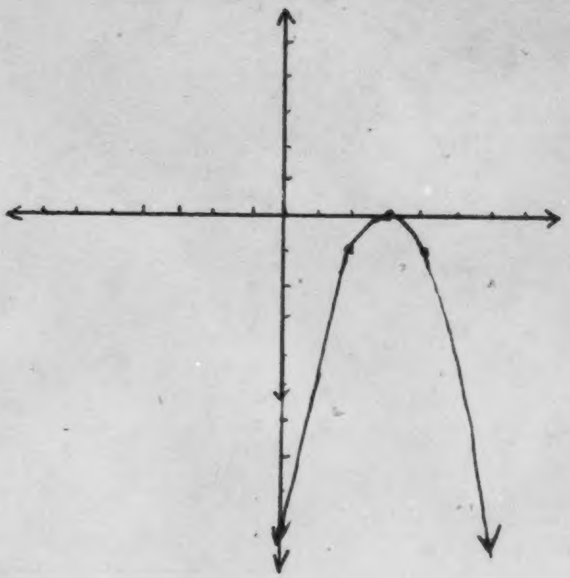
15. c)

x	$x^2 - 4$	y
0	$0^2 - 4$	-4
1	$1^2 - 4$	-3
2	$2^2 - 4$	0
-1	$-1^2 - 4$	-3
-2	$(-2)^2 - 4$	0



d)

x	$x^2 + 6x - 9$	y
0	$-(0)^2 + 6(0) - 9$	-9
1	$-(1)^2 + 6(1) - 9$	-4
2	$-(2)^2 + 6(2) -99$	-1
3	$-(3)^2 + 6(3) - 9$	0
4	$-(4)^2 + 6(4) - 9$	-1
5	$-(5)^2 + 6(5) - 9$	-4



UNIT VI - SYSTEMS OF OPEN SENTENCES

PURPOSE

This unit provides the students with alternative methods for solving systems of two equations in two variables. The techniques involved necessitate the integration of most concepts contained in the first semester of the course.

OVERVIEW

Students draw upon their knowledge of graphing techniques to solve systems of equations. Systematic algebraic manipulations provide alternative methods for finding the solutions to systems of open sentences. Word problems that involve the use of two variable solutions are discussed.

SUGGESTIONS TO THE TEACHER

Instructional Days: 9-12
Minimal Course Objectives: #2-5
Average Course Objectives: #1-7
Maximal Course Objectives: ALL

Emphasize the availability of several alternative methods for solving a system of open sentences, and stress the selection of the most efficient method.

Linear programming is a valuable topic for advanced students. It involves the utilization of algebraic skills in the solution of applied problems.

The Gauss Elimination Method for solving systems of equations may be a good strategy for reviewing operations with fractions and directed numbers.

VOCABULARY

coinciding lines
intersecting lines
linear programming
parallel lines
system of inequalities
system of simultaneous equations

UNIT VI - SYSTEMS OF OPEN SENTENCES

ENTERING PERFORMANCE OBJECTIVES

1. Determine the LCM of two arithmetic numbers.
2. Graph the equation of a line.
3. Write an equation in the slope-intercept form.
4. Determine whether a given point satisfies the equation of a line.
5. Write an equivalent equation for a given equation by applying the multiplication property of equality.

Assessment Tasks

1. a) The lowest common multiple of 12 and 16 is:

A. 4

B. 24

C. 32

D. 48

Answer ________

b) The lowest common multiple of 6, 9, and 15 is:

A. 15

B. 30

C. 90

D. None of the above

Answer ________

c) Find the lowest common multiple of 45 and 50.

Answer ________

d) Find the lowest common multiple of 8, 12, and 20.

Answer ________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. Graph the lines represented by the following equations:

a) $x + y = -4$

b) $y = \frac{2}{3}x - 5$

c) $y = 3x + 1$

d) $y = -\frac{4}{3}x + 2$

3. a) Which of the following is the slope-intercept form of $2x + 3y = -6$?

A. $2x = -3y - 6$

B. $x = -\frac{3}{2}y + 3$

C. $y = -\frac{2}{3}x - 2$

D. $y = \frac{3}{2}x - 2$

Answer __________

b) Which of the following is the slope-intercept form of $x + 2y = 4$

A. $x = \frac{1}{2}y + 2$

B. $x = -2y + 4$

C. $y = 2x + 4$

D. $y = -\frac{1}{2}x + 2$

Answer __________

c) Write the equation $3y - 4x = 6$ in slope-intercept form.

Answer __________

d) Write the equation $3y + x = -6$ in slope-intercept form.

Answer __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

4. a) Which of the following equations does the ordered pair (3,4) satisfy?

   A. $3x + 2y = 6$

   B. $x + y = -5$

   C. $\frac{2}{3}x = \frac{1}{2}y$

   D. $x - 2y = -12$

   Answer ___________

   b) Which of the following points satisfy the equation $3x - 5 = y$?

   A. (2,1)

   B. (1,2)

   C. (-1,2)

   D. (-1,-2)

   Answer ___________

   c) Does the point (-2,1) satisfy the equation $\frac{1}{2}x + 7y - 6 = 0$?

   Answer ___________

   d) Does the point $(\frac{1}{2},3)$ satisfy the equation $6x - y = 3$?

   Answer ___________

5. a) Which of the following is not an equivalent equation for $3x - 4y = -1$?

   A. $-6x + 8y = 2$

   B. $-3x = 4y + 1$

   C. $9x + 12y = -3$

   D. $-3x + 4y = 1$

   b) Multiply the equation $3y - x = 4$ by -4 to obtain an equivalent equation.

   Answer ___________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

5. c) Which of the following is an equivalent equation for $2x + 3y = 0$?

A. $4x + 6y = 2$

B. $-2x - 3y = -1$

C. $-6x - 9y = 0$

D. $8x - 12y = 0$

Answer ____________

d) Multiply the equation $5y - 2x = 3$ by -2 to obtain an equivalent equation.

Answer ____________

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UNIT VI - SYSTEMS OF OPEN SENTENCES

ENTERING PERFORMANCE OBJECTIVES

Answers

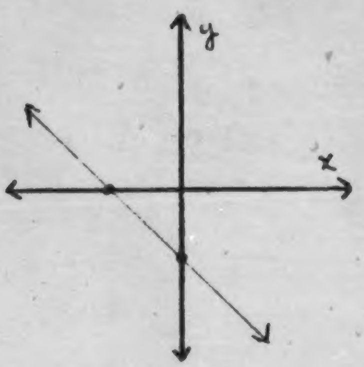
1. a) D

   b) C

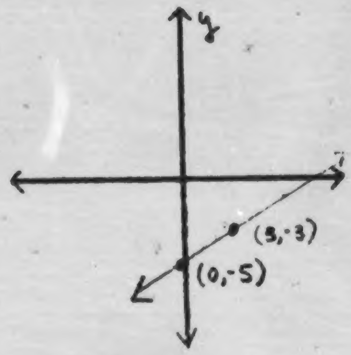
   c) 450

   d) 120

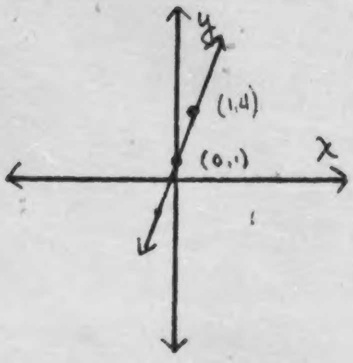
2. a)



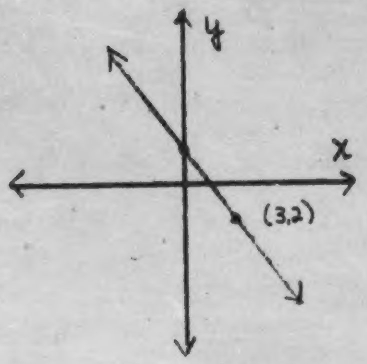
b)



c)



2. d)



3. a) C

   b) D

   c) $y = \frac{4}{3}x + 2$

   d) $y = -\frac{1}{3}x - 2$

4. a) C

   b) A

   c) yes

   d) no

5. a) B

   b) $-12y + 4x = -16$

   c) C

   d) $-10y + 4x = -6$

UNIT VI - SYSTEMS OF OPEN SENTENCES

PERFORMANCE OBJECTIVES

1. Determine whether the graphs of a system of equations are parallel, intersecting, or coinciding lines by examining slopes and y-intercepts. (II)
2. Solve a system of equations graphically. (III)
3. Solve a system of equations by the addition (subtraction) method. (III)
4. Solve a system of equations using multiplication with the addition (subtraction) method. (III)
5. Solve a system of equations by the substitution method. (III)
6. Solve word problems involving two variables and a system of equations. (III)
7. Solve a system of linear inequalities graphically. (III)

*8. Solve linear programming problems. (IV)

Minimal	Average	Maximal
# 2-5	#1-7	ALL

KEY SKILLS FOR END-OF-COURSE TESTING

13. Solve a system of equations in two variables.
14. Solve word problems involving two variables and a system of equations.

UNIT VI - SYSTEMS OF OPEN SENTENCES

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	---	251-254	425-426	354	225-226	---	222-223	308-311
2	337-339	251-253	422-423	389	243-245	180-183	247-251	312-314
3	346-347	255-257	429-431	396	251-255	184-186	252-255	318-320
4	350-352	258-259	432-434	396-399	252-255	188-192	256-259	318-320
5	340-342	261-262	427-428	404-406	246-249	193-195	261-263	315-317
6	344-345,355- 347-349,359 352-353	264-272	424,431 434-444	407-410	256-269	196-203	256-260 270-275	322-326 327-330
7	360-363	274-275	445-446	---	501-503	208-210	264-270	331-332
8	---	276-278	---	---	---	211	---	---

UNIT VI - SYSTEMS OF OPEN SENTENCES

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
1	308-314	---	274-277	321-326	101-105
2	308-211	397	274-277	323	104-105
3	318-320	403	281-283	331-332	107-108
4	322-324	403	281-283	334	109
5	325-326	400	278-280	328	106
6	315-317 320-321 326-327,324	406,409 412,413	296-315	341-349	111
7	328-332	---	388-394	---	117-124
8	---	---	---	---	---

PERFORMANCE OBJECTIVE VI-1

> Determine whether the graphs of a system of equations are parallel, intersecting, or coinciding lines by examining slopes and y-intercepts.

Tell whether the graphs of each of the following systems of equations are parallel lines, intersecting lines, or coinciding lines

a) $y = 2x + 3$ a) __________

$y = -5x + 11$

b) $y = 2x + 7$ b) __________

$y = 2x - 3$

c) $y = \frac{1}{2}x + 3$ c) __________

$2y = x + 6$

d) Which of the following equations has a graph which coincides with the graph of the equation $y = 3x - 4$?

A. $y = \frac{1}{3}x - 4$

B. $2y - 6x + 8 = 0$

C. $2y = 6x + 8$

D. None of the above

Answer: __________

PERFORMANCE OBJECTIVE VI-2

Solve a system of equations graphically.

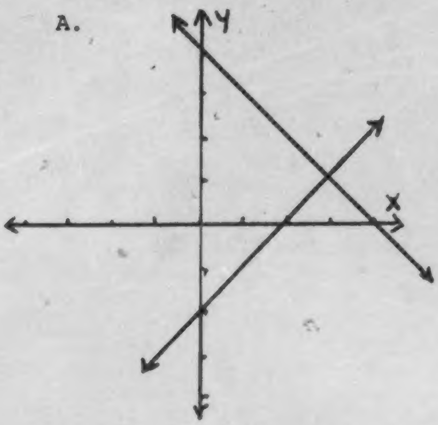
1. a) Match each system of equations with its graph.

______ 1) $y = x + 2;\ y - x = 4$

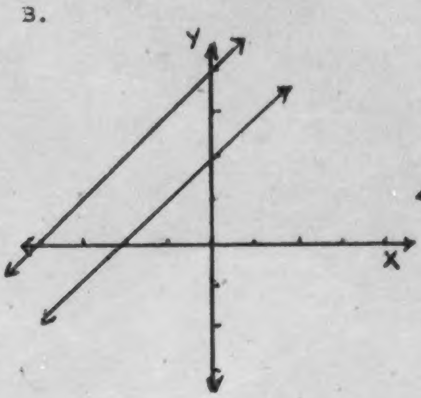
______ 2) $x - y = 2;\ x + y = 4$

______ 3) $x + y = 1;\ 2y = -2x + 2$

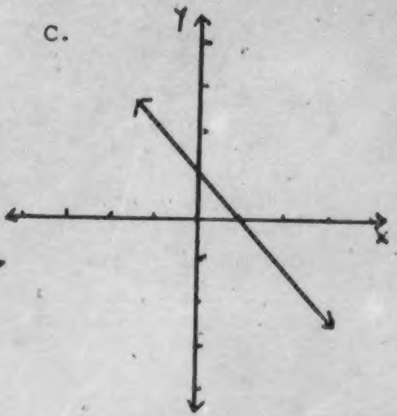
A.



B.



C.



b) Determine the solution set of the following systems of equations by graphing.

1) $x + 2y = 14$

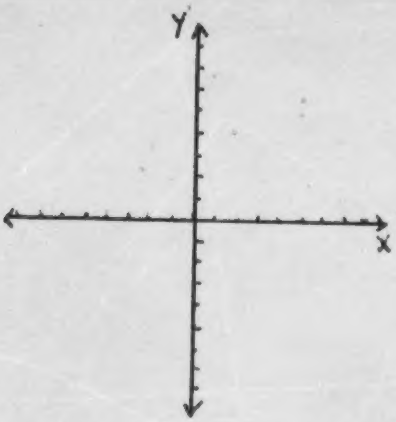
$y - 2x = 2$

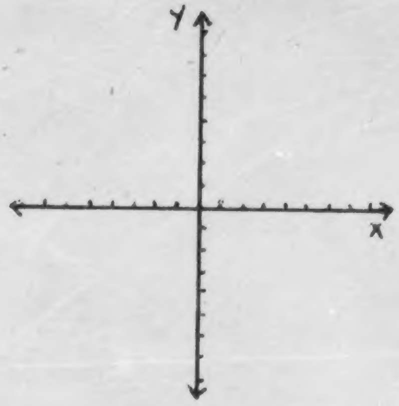
2) $2x + 3y = 9$

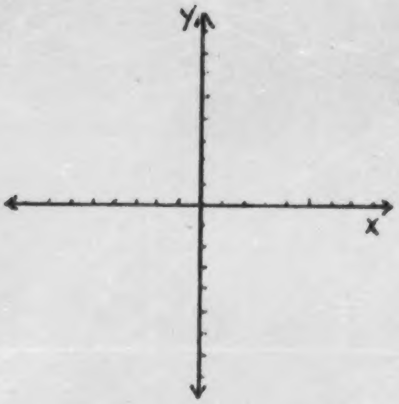
$y = x - 7$

3) $x + y = 5$

$y = 3 - x$







PERFORMANCE OBJECTIVE VI-3

Solve a system of equations by the addition (subtraction) method.

1. Solve simultaneously by the addition (subtraction) elimination method.

a) $3r + 2s = 10$

$5r + 2s = 10$

Answer: ___________

b) $3x + 2y = 12$

$-3x - 2y = 6$

Answer: ___________

c) $2x + 3y = 15$

$-2x + 9y = 21$

Answer: ___________

d) $-x + 2y = 6$

$2y - x = 6$

Answer: ___________

PERFORMANCE OBJECTIVE VI-4

Solve a system of equations using multiplication with the addition (subtraction) method.

1. Solve, using multiplication with the addition method.

a) $5p + 2q = 5$

$15p - 3q = 15$

Answer: ___________

b) $17r + 12s = -19$

$13r - 3s = 22$

Answer: ___________

c) $3a + 2b = 10$

$2a + 3b = -5$

Answer: ___________

d) $5p - 2q = 16$

$3p + 5q = 22$

Answer: ___________

PERFORMANCE OBJECTIVE VI-5

Solve a system of equations by the substitution method.

1. Solve the following systems of equation by substitution

a) $x + 2y = 5$

$3x + 2y = 3$

Answer: ________

b) $4p + q = 10$

$2p - 3q = 12$

Answer: ________

c) $2x + 3y = 2$

$4x - y = 18$

Answer: ________

d) $y = \frac{2}{3}x + 6$

$3y - 2x = 8$

Answer: ________

PERFORMANCE OBJECTIVE VI-6

Solve word problems involving two variables and a system of equations.

1. Solve each word problem:

   a) Elaine bought eight fuzzy peaches and one cucumber for $2.43. Les bought eight cucumbers and one fuzzy peach for $11.25. Find the cost of a cucumber.

   Answer: __________

   b) The sum of the digits of a two-digit number is 13. The number with digits interchanged is 14 more than 20 times the original tens digit. Find the original number.

   Answer: __________

   c) A canoeist who took 3 hours to paddle 9 miles upstream was able to return to his starting point in 30 minutes. At what rate could he paddle in still water?

   Answer: __________

   d) Eleven years ago Mrs. Alva was three times as old as her daughter Rose. Five years from now Mrs. Alva will be 5 years less than twice as old as Rose. How old is each now?

   Answer: __________

PERFORMANCE OBJECTIVE VI-7

Solve a system of linear inequalities graphically.

1. Solve each system of inequalities graphically

   a) $x + y \geq 6$

      $y - x > -8$

      Answer: __________

   b) $x + y < 5$

      $2y - x \geq -8$

      Answer: __________

   c) $x \leq -4$

      $x - 2y \geq -2$

      Answer: __________

   d) $y > -6$

      $2y + 2 < -x$

      Answer: __________

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PERFORMANCE OBJECTIVE VI-8

Solve linear programming problems.

1. Solve each problem.

   a) A regional park commission hopes to build campsites in two locations. At one location in the mountains, each campsite costs $750.00 and requires 36 man-hours of labor. At the other location, nearer a major city, each campsite costs $1500.00 and requires 12 man-hours of labor. The commission can spend at most $150,000.00 and use at most 4800 man-hours of labor in building the campsites. How many campsites should be built at each location in order to maximize the number of sites available?

   Answer: __________

   b) An automobile assembly plant has a maximum production capacity of 1200 cars per day. The sales force predicts it can sell at most 600 Cloudstars and 800 Dachsunds per day. The profit for a Cloudstar is $650.00 and the profit for a Dachsund is $500.00. How many of each car should be produced in order to make the greatest profit?

   Answer: __________

PERFORMANCE OBJECTIVE VI-8 (continued)

Solve linear programming problems.

c) A student gets a job in a Tastee-Freez stand. Each day he has 80 units of milk, 70 units of ice cream, and 30 units of syrup. The specialty of the house is the "Last Stand" which consists of 2 units of milk, 3 units of ice cream and 1 unit of syrup. Another speciality, the "Vanilla Plains," consists of 2 units of milk, 1 unit of ice cream, and 1 unit of syrup. The "Last Stand" sells for $.80 and the "Vanilla Plains" sells for $.50. How many sales of each would be most profitable?

Answer: __________

d) Each week the McKay Trucking Co. needs at least 650 gallons of diesel fuel, 324 gallons of gasoline, and 48 gallons of oil to keep its fleet of trucks in operation. Pacific Petroleum Co. can deliver 130 gallons of diesel fuel, 36 gallons of gasoline, and 4 gallons of oil for a wholesale rate of $60.00. A similar plan costing $75.00 is available from Overseas Oil Company for 65 gallons of diesel fuel, 54 gallons of gas, and 12 gallons of oil. How many standing orders should the McKay Co. place with each firm in order to meet its petroleum needs at the smallest cost?

Answer: __________

UNIT VI - SYSTEMS OF OPEN SENTENCES

Answers

VI-1

a) intersecting

b) parallel

c) coinciding

d) B

VI-2

a)

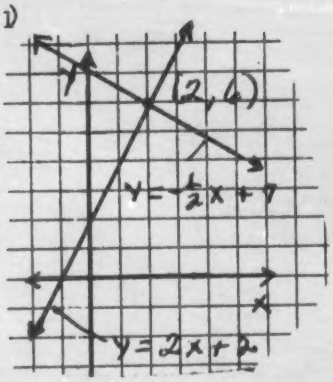
1) B

2) A

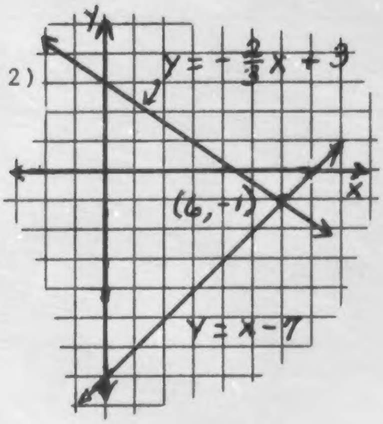
3) C

b)

1)

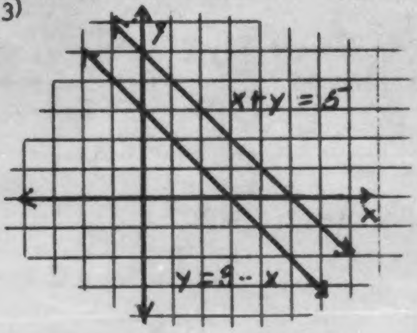


2)



VI-2 (continued)

3)



VI-3

a) $r = 0$
$s = 5$

b) $\emptyset$

c) $y = 3$

$x = 3$

d) {All x, y such that $-x + 2y = 6$}

VI-4

a) $p = 1$

$q = 0$

b) $r = 1$

$s = -3$

c) $a = 8$

$b = -7$

d) $p = 4$

$q = 2$

VI-5

a) $x = -1$

$y = -3$

b) $p = 3$
$q = -2$

c) $x = 4$

$y = -2$

d) $\emptyset$

UNIT VI - SYSTEMS OF OPEN SENTENCES

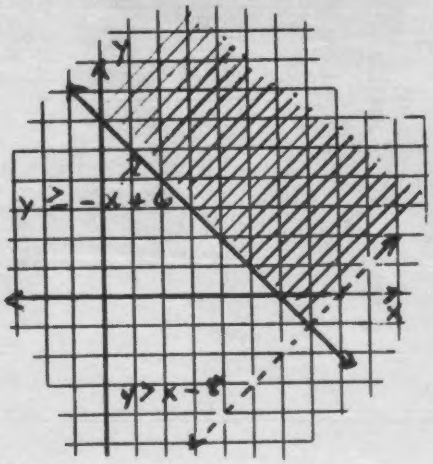
VI-6

a) A cucumber cost \$1.39,

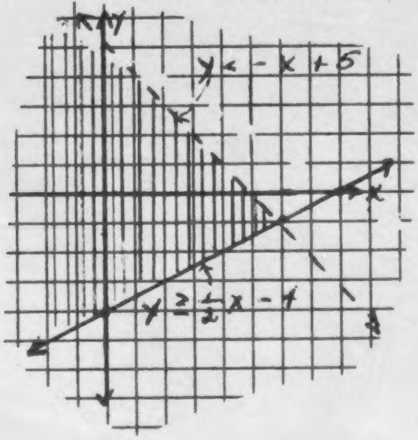
b) Original number is 49.

VI-7

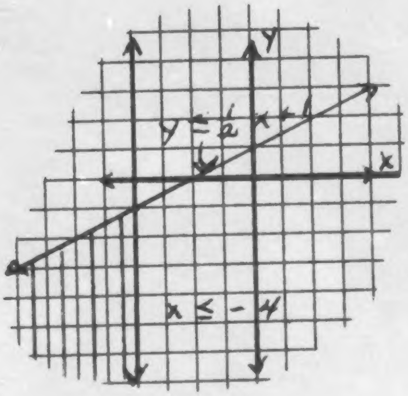
a)



b)



c)



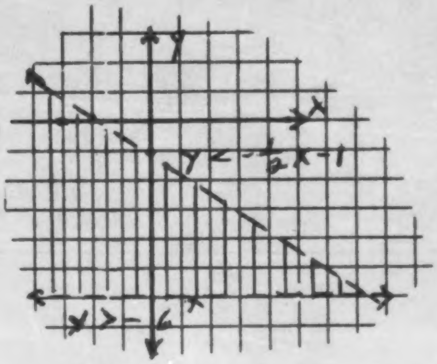
c) Canoeist could paddle $10\frac{1}{2}$ mph in still water,

d) Mrs. Alva is 44.

Rose is 22.

VI-7 (continued)

d)



VI-8

a) Let x = number of campsites in the mountains

y = number of campsites near the city

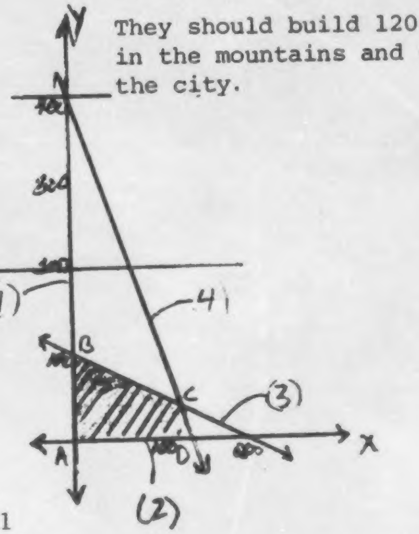
$x \geq 0$

$y \geq 0$

$750x + 1500y \leq 150,000$

$36x + 12y \leq 4800$

They should build 120 campsites in the mountains and 40 near the city.



Answers (continued)

VI-8 (continued)

b) Let x = number of Cloudstars produced each day
y = number of Dachsunds produced each day

(1) $x \le 600$

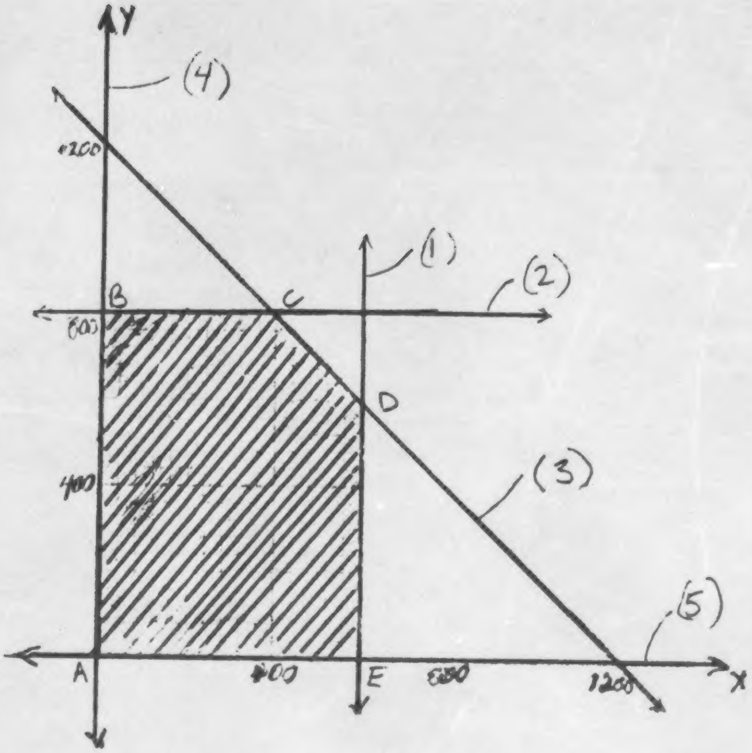
(2) $y \le 800$

(3) $x + y \le 1200$

(4) $x \ge 0$

(5) $y \ge 0$

To maximize profit, 600 of each car should be built each day.



c) Let x = the number of "Last Stands"
y = the number of "Vanilla Plains"

(1) $x \ge 0$

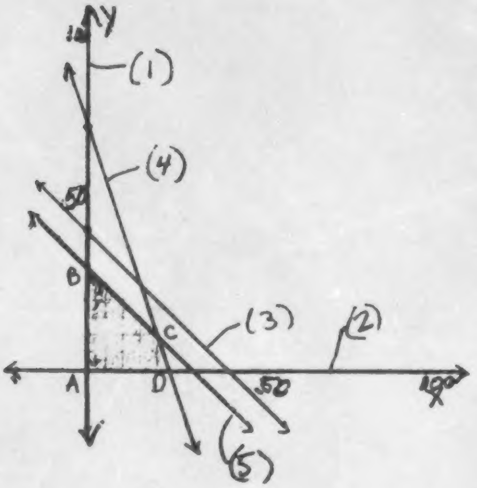
(2) $y \ge 0$

(3) $2x + 2y \le 80$

(4) $3x + y \le 70$

(5) $x + y \le 30$

He should sell 20 "Last Stands" and 10 "Vanilla Plains" to make the greatest profit.



Answers (continued)

VI-8 (continued)

d) Let x = number of orders from Pacific Petroleum

Let y = number of orders from Overseas Oil

(1) $x \geq 0$

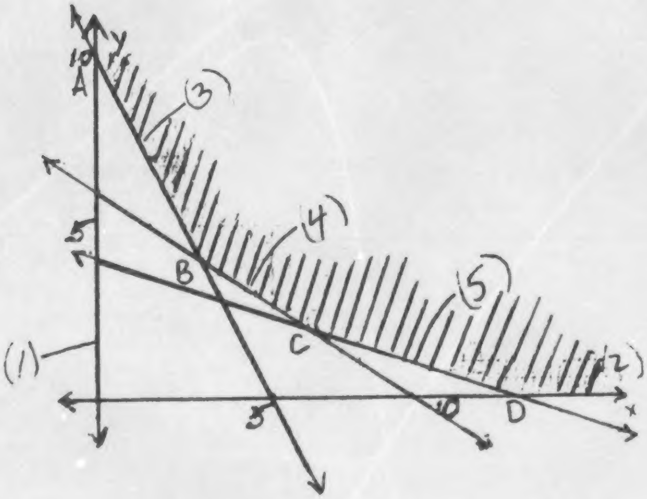
(2) $y \geq 0$

(3) $130x + 65y \geq 650$

(4) $36x + 54y \geq 324$

(5) $4x + 12y \geq 48$

To meet their needs at the smallest cost, McKay Trucking Co. should place 3 standing orders with Pacific Petroleum Co. and 4 with Overseas Oil Co.



UNIT VII - POLYNOMIALS

PURPOSE

This unit introduces the students to higher-order, non-linear relations. It is one of three interrelated units which concentrates on positive integral exponents. Students apply the laws of exponents to simplify polynomial expressions.

OVERVIEW

Students are introduced to the basic terminology of polynomials. The concept of combining similar terms is expanded to include the addition and subtraction of polynomials. The laws of exponents are combined with the addition and subtraction properties to simplify polynomial expressions. Some basic applications of polynomials are investigated through the solution of open sentences and word problems.

SUGGESTIONS TO THE TEACHER

Instructional Days: 16-19
Minimal Course Objectives: #1-12, 16-19
Average Course Objectives: #1-14, 16-19, 21, 22
Maximal Course Objectives: ALL

Due to the length of this unit, it is suggested that at least two tests be given.

Although many books discuss special products in relation to factoring, they are included as part of the multiplication objectives in this unit.

The FOIL method (First, Outer, Inner, Last) may be introduced as a procedure for multiplying two binomials.

Objectives containing the phrase "by inspection" are intended to require students to simplify expressions quickly without paper and pencil.

Some teachers feel that this unit is the most appropriate place to introduce zero and negative exponents.

Advanced students should be exposed to the most difficult exercises throughout this unit.

VOCABULARY

binomial
decreasing order
degree of a monomial in a variable
degree of a monomial
degree of a polynomial
increasing order
monomial
polynomial
simple form
trinomial

UNIT VII - POLYNOMIALS

ENTERING PERFORMANCE OBJECTIVES

1. Simplify arithmetic expressions containing exponents.
2. Add, subtract, multiply, and divide integers.
3. Compute long division problems.
4. Compute three place multiplication problems.
5. Simplify expressions using the distributive property.
6. Combine similar terms.
7. Solve equations.

Assessment Tasks

1. Simplify each of the following expressions:

a) $4 \cdot 3^2 \cdot 4^2 \cdot 3^2$ a) __________

b) $\frac{2^5}{2^3}$ b) __________

c) $5^2 + 9^2$ c) __________

d) $1^5 + 2^5 - 1^{10}$ d) __________

2. a) $4 + (-6) + (-7)$ a) __________

b) $814 + (-857) + 311$ b) __________

c) $1.033 + (-0.1) + (-10.066)$ c) __________

d) $1\frac{1}{3} + (-2\frac{3}{4})$ d) __________

e) $308 - 684$ e) __________

f) $-177 - -643$ f) __________

g) $-20.5 - 856.175$ g) __________

h) $11\frac{1}{3} - 12\frac{2}{5}$ h) __________

i) $(96)(-65)$ i) __________

j) $(-1.3)(-3.5)$ j) __________

k) $(-56)(-79)$ k) __________

l) $(-3\frac{6}{7})(2\frac{1}{3})$ l) __________

m) $(-208) \div (-16)$ m) __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. (continued)

n) $21.707 \div (-.07)$ n) ____________

o) $(-6240) \div 65$ o) ____________

p) $(-1\frac{3}{8}) \div (-5\frac{1}{2})$ p) ____________

3. a) $58\overline{)21{,}808}$ a) ____________

b) $72\overline{)60{,}048}$ b) ____________

c) $29\overline{)16{,}625}$ c) ____________

d) $126\overline{)38{,}430}$ d) ____________

4.

a)	257	b)	592	c)	728	d)	364
	x917		x795		x238		x419
a)	____________	b)	____________	c)	____________	d)	____________

5. Simplify each of the following expressions:

a) $7(x + 4 - 6y)$ a) ____________

b) $-5(3x - 9 + 7y)$ b) ____________

c) $8(9x + 5y - 8)$ c) ____________

d) $-1(2x^2 - 3x - 9)$ d) ____________

6. Combine similar terms.

a) $23 + 5t + 7y + t + y + 27$ a) ____________

b) $6x^4 + 3x^3 - 1 + 4x^4 - 2x^3 + 5$ b) ____________

c) $5m^3 - 6m^2 + 4m - 6m^3 + 6m^2 - 5m$ c) ____________

d) $2x^2 - 6x - 15 - 11x^2 + 7x - 6$ d) ____________

7. Solve each of the following equations:

a) $7(y + 2) + 4(y - 1) + 12 = 0$ a) ____________

b) $3(8n - 5) - 3(1 - n) = 9$ b) ____________

c) $2 - 7(m - 1) = 3(m - 2) - 5(m + 3)$ c) ____________

d) $6(2x + 1) - 3(4x - 3) - (6x + 10) =$
$-(4x - 3) + 3$ d) ____________

UNIT VII - POLYNOMIALS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) 5184
b) 4
c) 106
d) 32

2. a) -9
b) 268
c) -9.133
d) $-1\frac{5}{12}$
e) -376
f) 466
g) -876.675
h) $-1\frac{1}{15}$
i) -6240
j) 4.55
k) 4424
l) -9
m) 13
n) -310.1
o) -96
p) $\frac{1}{4}$

3. a) 376
b) 834
c) $573\frac{8}{28}$
d) 305

4. a) 235,669
b) 470,640
c) 173,264
d) 152,516

5. a) $7x + 28 - 42y$
b) $-15x + 45 - 35y$
c) $72x + 40y - 64$
d) $-2x^2 + 3x + 9$

6. a) $6t + 8y + 50$
b) $10x^4 + x^3 + 4$
c) $-m^3 - m$
d) $-9x^2 + x - 21$

7. a) $y = -2$
b) $n = 1$
c) $m = 6$
d) $x = -\frac{1}{2}$

UNIT VII - POLYNOMIALS

PERFORMANCE OBJECTIVES

1. Classify a polynomial according to the number of terms: monomial, binomial, trinomial. (II)
2. State the degree of a monomial. (II)
3. State the degree of a polynomial. (II)
4. Arrange a polynomial in increasing or decreasing order.
5. Add polynomials. (II)
6. Subtract polynomials. (II)
7. Multiply a monomial by a monomial. (II)
8. Determine the power of a product. (II)
9. Simplify expressions involving powers of products, multiplication of monomials, and combining similar terms. (II)
10. Multiply a polynomial by a monomial. (II)
11. Multiply two binomials. (II)
12. Multiply two binomials of the form $(a + b)(a - b)$ by inspection.
13. Find the square of a binomial by inspection. (II)
14. Multiply a trinomial by a binomial. (II)
15. Multiply two polynomials each containing at least three terms. (II)
16. Solve open sentences involving operations with polynomials. (III)
17. Divide a monomial by a monomial. (II)
18. Divide a polynomial by a monomial. (II)
19. Simplify expressions that involve dividing by a monomial and combining similar terms. (II)
20. Simplify expressions involving zero and negative exponents. (II)
21. Divide a polynomial by a polynomial. (II)
22. Solve word problems involving polynomials. (III)

Minimal	Average	Minimal
#1 - 12, 16 - 19	#1 - 14, 16 - 19, 21, 22	All

KEY SKILLS FOR END-OF-COURSE TESTING

15. Add and subtract polynomials.

16. Multiply polynomials.

17. Divide polynomials.

18. Solve linear equations involving polynomials.

UNIT VII - POLYNOMIALS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	116-118	292-293	148	283	140-141	270-271	293	200-201
2	116-118	294	148	296	138-140 293-295	---	---	197
3	116-118	295	148-150	296	138-140 293-295	---	293	200-201
4	---	296	149-150	---	138-140 294-295	---	---	--
5	116-118	295	151-152	286	142-143 298-300	270-271	289-291	208-211
6	116-118	295	153-155	288	144-147 298-300	270-271	289-291	212-214
7	120-121	298	156-157	292	148-150	221	83-86	---
8	122-123	298	159-161	244-245	89-91	224	389-391	363-365
9	122-123	298	160-161	---	---	---	95	---
10	124-126	299-301	162-163	292-295	148-151 156-157	274	---	215 216
11	128-130 165-166	303-305	164-166	295	148-151 301-303 156-157	275,277	292-295	217,222
12	156-159	305	171	295	301-303 159-162	283-284	305-307	217, 222
13	160-162	304	167-171	295	301-303 159-162	280-281	308-309	221-222
14	130	305	---	295	152-154	---	---	217
15	130	---	---	295	152-154	---	---	218
16	118-119 125,126	305	---	491-493	313-314	310-311	---	---

UNIT VII - POLYNOMIALS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
17	145-147	339-340	241-243	245,425	352-355	227-229	91-93	---
18	151-153	339-340	---	---	352-355	227-229	335-336	348-351
19	153	342-344	259-261	461	---	227-229	---	---
20	148-149	349-352	---	238-239	84-87	230	387-391	359-362
21	220-222	342-344	254-255	---	352-355	306-308	337-340	352-356
22	131-133 125-126	324-329	172-177 223-228	---	---	311	---	356-357

UNIT VII - POLYNOMIALS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
1	316-317	68-69	438	189	---	---
2	---	---	---	215	169	169
3	318-319	128	---	215	169	169
4	318-319	68-69	---	189-190	134	134
5	323-328	70-73	439 443-444	192-194	---	135-136
6	330-336	74-77	439 445-446	192-194	---	135-136
7	346-349	79-82	447	186-188	---	148
8	350-351	79-82	---	130-132	148	148
9	348-353	79-82	---	130-132	151	151
10	352-355	84-85	---	---	151	151
11	356-359	86-91	449	195-198	152-157	152-157
12	360-362	---	---	---	152-157	152-157
13	360-362	---	---	---	155	155
14	356-359	86-88	449	196,198	157	157
15	358	90-91	---	198	158	158
16	354	76,85	---	426-440	159	159

UNIT VII - POLYNOMIALS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)	Johnson et al (Part II) (1977)
17	368-370	98-100	451	187-188	162	162
18	371-373	101-104	---	199	167	167
19	---	---	---	---	167	167
20	---	---	---	---	240	240
21	374-376	101-104	451	203-206	169-173	169-173
22	354 359 362	72-73,103- 77,88,104	---	444-446	---	---

PERFORMANCE OBJECTIVE VII-1

Classify a polynomial according to the number of terms; monomial, binomial, or trinomial.

Identify each polynomial as a monomial, binomial, or a trinomial.

a) $3r^2s + 15$

Answer ________

b) $x^3 - 2xy + y^2$

Answer ________

c) $4x^2y^3z^4$

Answer ________

d) $-5 + 2v^2w + 5x^3$

Answer ________

PERFORMANCE OBJECTIVE VII-2

State the degree of a monomial.

a) State the degree of $-3x^2y^3z$.

Answer ________

b) State the degree of $5x^3y$.

Answer ________

c) State the degree of $11x^3yz^5$.

Answer ________

d) The degree of $-5x^2y^7z$ is:

A. 9

B. 5

C. 14

D. 10

Answer ________

PERFORMANCE OBJECTIVE VII-3

State the degree of a polynomial.

1. State the degree of each of the following:

a) $4x^2yz + 7x^3z + 13x^5y^2z^2 + 17x^2y$

Answer __________

b) $5x^3yz^4 + 7x^5yz + 8xyz + 9x^2y^2z^2$

Answer __________

c) $6x^2y^2z^3 + 9xyz^3 + 11x^4y^4z^3 + 8x^3yz.$

Answer __________

d) $7y^2x^2z + 15z^2y^2x^5 + 11x^3z^8y + 9zxy$

Answer __________

e) The degree of $-15x^3yz^5 + 75x^2y^2z + 15xy^7z^{11}$ is:

A. 9

B. 5

C. 19

D. 11

Answer __________

PERFORMANCE OBJECTIVE VII-4

Arrange a polynomial in increasing or decreasing order.

a) Arrange the following polynomial in decreasing order of the degree of x:

$4xy^2z + 7x^3z + 13x^5y^2z + 17x^2y$

Answer __________

b) Arrange the following polynomial in increasing order of the degree of

y: $5x^3y^3z^4 + 7x^5yz + 8xy^4z + 9x^2y^2z^2$

Answer __________

c) Arrange the following polynomial in decreasing order of the degree of z:

$9xyz + 11x^3yz^8 + 7x^2y^2z^3 + 15x^5y^2z^2$

Answer __________

d) Arrange the following polynomial in increasing order of the degree of x:

$8x^3yz^2 + 13x^5z + 2x^2y^2z^2 + 11x^4yz$.

Answer __________

PERFORMANCE OBJECTIVE VII-5

Add polynomials.

Simplify:

a) $(x + 5) + (4x - 7) + (5x - 8)$

Answer __________

b) $(8x^3 - 40x^2 + 50x) + (-20x^2 + 10x - 125)$

Answer __________

c) $(5x - y + z) + (3x - 8 - y) + (3y + 7)$

Answer __________

d) $(y^2 + 4y + 6) + (y^2 - 4y - 12) + (2y + 6 - y^2)$

Answer __________

PERFORMANCE OBJECTIVE VII-6

Subtract polynomials.

Simplify:

a) $(5a - 6b + c) - (3a + b - c)$

Answer ___________

b) $(8ab - 5ax) - (9ax + 11ab)$

Answer ___________

c) $(a^2 - a + 4) - (5 + 2a - a^2)$

Answer ___________

d) $(3a^2 + 5ab - 7b^2) - (3ab - a^2 - b^2)$

Answer ___________

PERFORMANCE OBJECTIVE VII-7

Multiply a monomial by a monomial.

Find the product of each pair of monomials:

a) $(2a^4)\ (5a^2)$ Answer __________

b) $(-3m^2n)\ (7mn^2)$ Answer __________

c) $(2ax^2y)\ (-5a^4xy^3)$ Answer __________

d) $(-3a)\ (-2a^2b)\ (-a^3b^2)$ Answer __________

e) $(a^m)(a^n)$ Answer __________

PERFORMANCE OBJECTIVE VII-8

Determine the power of a product.

Simplify:

a) $(3a^2)^3$

Answer __________

b) $(-5m^4n^2)^3$

Answer __________

c) $(-3a^3b^4)^2$

Answer __________

d) $(2b^4c^5)^3$

Answer __________

e) $(a^mb)^n$

Answer __________

PERFORMANCE OBJECTIVE VII-9

Simplify expressions involving powers of products, multiplication of monomials, and combining similar terms.

Simplify:

a) $(3x^2y)^2 + (3x^2y^2)(2x^2)$

Answer ________

b) $(3u)(u^2v)^3 + (2u)^2(-u^5v^3)$

Answer ________

c) $(-6y^2z^3)^2(2yz) - (3yz^2)^3(-2y^2z)$

Answer ________

d) $(5m^2n)^2(3m^3) - (3m^2)^2(2m^3n^2)$

Answer ________

PERFORMANCE OBJECTIVE VII-10

Multiply a polynomial by a monomial.

Simplify:

a) $7x^2(x^3 - 2x^2 + 11)$

Answer __________

b) $4x(5 - x - 10x^2)$

Answer __________

c) $-2abc(5a + 2b - 3c + 8)$

Answer __________

d) $2xy(3x^2 - 8xy + 5y^2)$

Answer __________

PERFORMANCE OBJECTIVE VII-11

Multiply two binomials.

Simplify:

a) $(3x + 4)\ (2x + 5)$

Answer ____________

b) $(2x - 7)\ (x - 8)$

Answer ____________

c) $(3x - 2)\ (2x = 3)$

Answer ____________

d) $(-2x + 3)\ (3x - 2)$

Answer ____________

PERFORMANCE OBJECTIVE VII-12

Multiply two binomials of the form (a + b) (a - b) by inspection.

Find the product of the following binomials by inspection:

a) $(3x + 2)\ (3x - 2)$

Answer ___________

b) $(y^2 - 5)\ (y^2 + 5)$

Answer ___________

c) $(w - x)\ (w + x)$

Answer ___________

d) $(5x + \frac{3}{7})\ (5x - \frac{3}{7})$

Answer ___________

PERFORMANCE OBJECTIVE VII-13

Find the square of a binomial by inspection.

1. Find the square of the following binomials by inspection:

   a) $(x + 3)^2$

      Answer ________

   b) $(2x - 3)^2$

      Answer ________

   c) $(5w^2 + 6t^3)^2$

      Answer ________

   d) $(xy^2 - w^3v)^2$

      Answer ________

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PERFORMANCE OBJECTIVE VII-14

Multiply a trinomial by a binomial.

Find each product:

a) $$\begin{array}{r} m^2 + 5m - 6 \\ \underline{2m + 1} \end{array}$$

Answer ________

b) $$\begin{array}{r} 5x^2 - 6x + 7 \\ \underline{3x - 1} \end{array}$$

Answer ________

c) $(3c^2 - c + 5)\ (c^2 - 1)$

Answer ________

d) $(x^2 + 2xy + y^2)\ (x + y)$

Answer ________

PERFORMANCE OBJECTIVE VII-15

Multiply two polynomials each containing at least 3 terms.

Simplify the following:

a) $(x^2 + 3x - 2)(x^2 - 5x + 3)$

Answer ________

b) $(2x^2 - x + 7)(3x^2 + 2x - 5)$

Answer ________

c) $(3x^2 + 2xy - y^2)(2x^2 - 3xy + 5y^2)$

Answer ________

d) $(a + 2b + c)(2a + 3b - c)$

Answer ________

PERFORMANCE OBJECTIVE VII-16

Solve open sentences involving operations with polynomials.

Solve the following open sentences:

a) $-10(3 - 4n) - 7(5n + 3) = -51$

Answer __________

b) $6k - 5(3k + 2) = 5(k - 1) - 8$

Answer __________

c) $3(x - 2) + 4(x + 6) = 3(x + 14) - 2$

Answer __________

d) $4(2m + 5) - 2(5 - 6m) < 12 - 3(10 - 8m)$

Answer __________

PERFORMANCE OBJECTIVE VII-17

Divide a monomial by monomial.

Simplify:

a) $\dfrac{42x^8y^3w}{6x^5y^3}$

Answer ____________

b) $\dfrac{35a^4b^3c^2}{-5ab^5c}$

Answer ____________

c) $\dfrac{36m^6n}{45m^5n}$

Answer ____________

d) $-27a^7b^9c^5 \div -3a^5b^7c^8$

Answer ____________

e) $\dfrac{a^m}{a^n}$

Answer ____________

PERFORMANCE OBJECTIVE VII-18

Divide a polynomial by a monomial.

1. Simplify:

a) $\dfrac{-9a^2b + 15ab^2 - 21ab}{-3ab}$

Answer __________

b) $\dfrac{ax^3y - bxy^3 + cx^2y^2}{-x^2y^2}$

Answer __________

c) $(4a^2 - 6a^3 + 4a^4) \div (2a^2)$

Answer __________

d) $(3mx^3 + 18nx^3 - 12x^3y) \div 3x^3$

Answer __________

PERFORMANCE OBJECTIVE VII-19

Simplify expressions that involve dividing by a monomial and combining similar terms.

Simplify:

a) $\frac{38p^5q^5}{19\,pq^2} - \frac{15p^4q^4}{-3p^2q}$

Answer ___________

b) $\frac{25y^3 - 15y^2 + 30y}{-5y} + \frac{8y^5 - 3y^3}{y^3}$

Answer ___________

c) $\frac{40cd^2 - 32c^2d + 24c^2d^2}{-8cd} + \frac{24c^3d - 12c^2d^2}{3c^2d}$

Answer ___________

d) $\frac{12m^5n^4 - 18m^6n^5}{6mn^2} - \frac{9m^7n^3 + 3m^6n^2}{3m^2}$

Answer ___________

PERFORMANCE OBJECTIVE VII-20

Simplify expressions involving zero and negative exponents.

Simplify:

a) $\frac{-5x^{9}y^{-5}w^{-2}}{-75x^{-4}y^{-3}}$

Answer __________

b) $\frac{3^{-2}}{9^{-1}}\frac{x^{2}y^{0}}{x^{-2}y}$

Answer __________

c) $\frac{x^{-2}y^{0}z}{x^{0}y^{-1}z^{2}}$

Answer __________

d) $(x^{2}y^{0}z^{-3})^{-3}$

Answer __________

e) a^{-n}

Answer __________

PERFORMANCE OBJECTIVE VII-21

Divide a polynomial by a polynomial.

Simplify:

a) $x - 6 \overline{)\, x^3 - 12x + 25}$

b) $x + y \overline{)\, 2x^2 + 3xy + y^2}$

c) $\dfrac{x^3 - y^3}{x - y}$

Answer ____________

d) $\dfrac{10x^2 + 7 - 19x}{2x - 1}$

Answer ____________

PERFORMANCE OBJECTIVE VII-22

Solve word problems involving polynomials.

Solve each problem. Show all work.

a) The squares of two consecutive integers differ by 75. Find the integers.

Answer __________

b) A rectangular pool is 5 feet longer than it is wide. If a concrete walk 2 feet wide is placed around the pool, the area covered by the pool and the walk is 156 square feet greater than the area covered by the pool alone. What are the dimensions of the pool? (Hint: a sketch may help.)

Answer __________

c) A square and a rectangle have the same area. The length of the rectangle is three inches more than a side of the square. The width of the rectangle is 2 inches less than a side of the square. Find the side of the square. (Hint: a sketch may help.)

Answer __________

d) A rectangular piece of glass, whose dimensions are 17 cm by 12 cm, must be trimmed by cutting an equal amount from the length and width, so that its area is 104 cm^2. How much must be cut from the length and width?

Answer __________

Answers

VII-1

a) binomial

b) trinomial

c) monomial

d) trinomial

VII-2

a) 6

b) 4

c) 9

d) d

VII-3

a) 9

b) 8

c) 11

d) 12

e) c

VII-4

a) $13x^5y^2z + 7x^3z + 17x^2y + 4xy^2z$

b) $7x^5yz + 9x^2y^2z^2 + 5x^3y^3z^4 + 8xy^4z$

c) $11x^3yz^8 + 7x^2y^2z^3 + 15x^5y^2z^2 + 9xyz$

d) $2x^2y^2z^2 + 8x^3yz^2 + 11x^4yz + 13x^5z$

VII-5

a) $10x - 10$

b) $8x^3 - 60^2 + 60x - 125$

c) $8x + y + z - 1$

d) $y^2 + 2y$

VII-6

a) $2a - 7b + 2c$

b) $-3ab - 14ax$

c) $2a^2 - 3a - 1$

d) $4a^2 + 2ab - 6b^2$

VII-7

a) $10a^6$

b) $-21m^3n^3$

c) $-10a^5x^3y^4$

d) $-6a^6b^3$

e) $a^{m + n}$

VII-8

a) $27a^6$

b) $-125m^{12}n^6$

c) $9a^6b^8$

d) $8b^{12}c^5$

e) $a^{mn}b^n$

UNIT VII - POLYNOMIALS

Answers (continued)

VII-9

a) $15x^4y^2$

b) $-u^7v^3$

c) $126y^5z^7$

d) $57m^7n^2$

V-10

a) $7x^5 - 14x^4 + 77x^2$

b) $20x - 4x^2 - 40x^3$

c) $-10a^2bc - 4ab^2c + 6abc^2 - 16abc$

d) $6x^3y - 16x^2y^2 + 10xy^3$

VII-11

a) $6x^2 + 23x + 20$

b) $2x^2 - 23x + 56$

c) $6x^2 - 13x + 6$

d) $-6x^2 + 13x - 6$

VII-12

a) $9x^2 - 4$

b) $y^4 - 25$

c) $w^2 - x^2$

d) $25x^2 - \frac{9}{49}$

VII-13

a) $x^2 + 6x + 9$

b) $4x^2 - 12x + 9$

c) $25w^4 + 60w^2t^3 + 36t^6$

d) $x^2y^4 - 2w^3vxy + w^6v^2$

VII-14

a) $2m^3 + 11m^2 - 7m - 6$

b) $15x^3 - 23x^2 + 27x - 7$

c) $3c^4 - c^3 + 2c^2 + c - 5$

d) $x^3 + 3x^2y + 3xy^2 + y^3$

VII-15

a) $x^4 - 2x^3 - 14x^2 + 19x - 6$

b) $6x^4 + x^3 + 9x^2 + 19x - 35$

c) $6x^4 - 5x^3y + 7x^2y^2 + 13xy^3 - 5y^4$

d) $2a^2 + 7ab + 6b^2 + ac + bc - c^2$

VII-16

a) $n = 0$

b) $\frac{3}{14} = k$

c) $x = 5\frac{1}{2}$

d) $m > 7$

VII-17

a) $7x^3w$

b) $\frac{-7a^3c}{b^2}$

c) $\frac{4m}{5}$

d) $\frac{9a^2b^2}{c^3}$

e) a^{m-n}

Answers (continued)

VII-18

a) $3a - 5b + 7$

b) $\frac{-ax}{y} + \frac{by}{x} - c$

c) $2 - 3a + 2a^2$

d) $m + 6n - 4y$

VII-19

a) $7p^2q^3$

b) $3y^2 + 3y - 9$

c) $-9d + 12c - 3cd$

d) $m^4n^2 - 6m^5n^3$

VII-20

a) $\frac{x^{13}}{15y^2w^2}$

b) $\frac{x^4}{y}$

c) $\frac{y}{x^2z}$

d) $\frac{z^9}{x^6}$

e) $\frac{1}{a^n}$

VII-21

a) $x - 6 - \frac{11}{x - 6}$

b) $2x + y$

c) $x^2 + xy + y^2$

d) $5x - 7$

VII-22

a) 37, 38

b) 15' x 20'

c) Side = 6

d) 4 cm

UNIT VIII - FACTORING

PURPOSE

The concepts contained in this unit provide the students with a technique for expressing polynomials as indicated products. When polynomials are written in this form, the division property of equality can be used to simplify algebraic fractions and the zero product rule can be used to solve higher order equations.

OVERVIEW

Algebraic factoring is introduced through a review of the concept of prime factorization. Various techniques for factoring polynomials are presented and applied to the solution of quadratic equations and word problems.

SUGGESTIONS TO THE TEACHER

Instructional Days: 12-15
Minimal Course Objectives: 1-8
Average Course Objectives: ALL
Maximal Course Objectives: ALL

An alternative method for factoring quadratic trinomials is offered:

> A trinomial of the form $ax^2 + bx + c$ can be expressed as $ax^2 + rx + sx + c$ where $r + s = b$ and $rs = ac$.
>
> The trinomial $6x^2 + 19x - 7$ can be written $6x^2 - 2x + 21x - 7$ where $-2 + 21 = 19$ and $-2 \cdot 21 = 6 \cdot 7. = -42$.
>
> To complete the factoring, $6x^2 - 2x + 21x - 7 = 2x(3x - 1) + 7(3x - 1) = (3x - 1)(2x + 7)$.

The following procedure could be used for general factoring of polynomials:

1. Check for common factors.
2. Check for difference of squares.
3. Check for a trinomial square.
4. Factor as a product of sums, a product of differences, or a product of a sum and a difference.
5. Check each factor to see whether it can be factored by one of the methods in steps #2 - 4.

VOCABULARY

- binomial difference
- binomial sum
- composite
- constant term
- cubic equation
- difference of squares
- factor
- greatest common factor
- linear term
- polynomial equation
- prime factorization
- prime number
- product
- quadratic equation
- quadratic term
- trinomial square
- zero product property

UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

1. Identify prime and composite numbers.
2. Prime factor given whole numbers.
3. Determine the greatest common factor of two or more whole numbers.
4. Multiply a polynomial by a monomial.
5. Multiply a binomial sum by a binomial difference.
6. Square a binomial.
7. Solve equations.

Assessment Tasks

1. a) Identify each of the following as being prime or composite.

A.	17	A.	__________
B.	26	B.	__________
C.	13	C.	__________
D.	68	[illegible]	__________
E.	57	E.	__________

b) Which of the following is a prime number?

A. 0

B. 1

C. 19

D. 87

Answer __________

c) Which of the following is a composite number?

A. 5

B. 11

C. 20

D. 53

Answer __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. a) $2^3 \cdot 5$ is the prime factorization of which of the following numbers?

A. 30

B. 40

C. 1000

D. None of the above.

Answer ___________

b) The prime factorization of 72 is:

A. $2 \cdot 3 \cdot 12$

B. $8 \cdot 9$

C. $2^4 \cdot 3^2$

D. $2^3 \cdot 3^2$

Answer ___________

c) The prime factorization of 300 is:

A. $2 \cdot 3^3 \cdot 5^2$

B. $2^3 \cdot 2^2 \cdot 5$

C. $2^2 \cdot 3 \cdot 5^2$

D. None of the above.

Answer ___________

d) Prime factor each of the following numbers:

A. 36 A. ___________

B. 54 B. ___________

C. 84 C. ___________

D. 105 D. ___________

E. 140 E. ___________

F. 180 F. ___________

UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

3. a) What is the greatest common factor of 24 and 36?

   Answer __________

   b) The greatest common factor of 12 and 16 is:

   A. 2

   B. 4

   C. 6

   D. 8

   Answer __________

   c) What is the greatest common factor of 15, 25, and 30?

   Answer __________

   d) The greatest common factor of 30, 54, and 72 is:

   A. 6

   B. 9

   C. 12

   D. 27

   Answer __________

4. Simplify each of the following:

   a) $-8x\ (3x^2 - 5x - 1)$ a) __________

   b) $2x^3y\ (-8x^2y + 11xy^2)$ b) __________

   c) $a^4\ (a^3 - 2a^2 + 2)$ c) __________

   d) $5s^2t\ (2t^2 - 3s^2t^2 + 6s^2)$ d) __________

5. a) $(8x - 7y)(8x + 7y)$ a) __________

   b) $(xy + 9)(xy - 9)$ b) __________

   c) $(x + 3)(x - 3)$ c) __________

   d) $(3x - 1)(3x + 1)$ d) __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

6. a) Which of the following is the square of $(7x + 8y)$?

A. $49x^2 + 56xy + 64y^2$

B. $49x^2 + 56xy + 64y$

C. $49x^2 + 112xy + 64y^2$

D. $49x^2 + 64y^2$

Answer ___________

b) Which of the following is the square of $(9x - 4y)$?

A. $81x^2 + 72xy - 16y^2$

B. $81x^2 - 72xy + 16y^2$

C. $81x^2 - 16y^2$

D. $9x^2 - 13xy + 16y^2$

Answer ___________

c) $(3x - 1)^2$

Answer ___________

d) $(2x + 9)^2$

Answer ___________

7. Solve each of the following equations:

a) $\frac{1}{2}x + 6 = 0$ a) ___________

b) $\frac{2}{3}x = 0$ b) ___________

c) $5t + 9 - (3t + 1) = 0$ c) ___________

d) $3(5x - 7) + 66 = 0$ d) ___________

UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) A. prime

B. composite

C. prime

D. composite

E. composite

b) C

c) C

2. a) B

b) D

c) C

d) A. $2^2 \cdot 3^2$

B. $2 \cdot 3^3$

C. $2^2 \cdot 3 \cdot 7$

D. $3 \cdot 5 \cdot 7$

E. $2^2 \cdot 5 \cdot 7$

F. $2^2 \cdot 3^2 \cdot 5$

3. a) 12

b) B

c) 5

d) A

4. a) $-24x^3 + 40x^2 + 8x$

b) $-16x^5y^2 + 22x^4y^3$

c) $a^7 - 2a^6 + 2a^4$

d) $10s^2t^3 - 15s^4t^3 + 30s^4t$

5. a) $64x^2 - 49y^2$

b) $x^2y^2 - 81$

c) $x^2 - 9$

d) $9x^2 - 1$

6. a) C

b) B

c) $9x^2 - 6x + 1$

d) $4x^2 + 36x + 81$

7. a) $x = -12$

b) $x = 0$

c) $t = -4$

d) $x = -3$

UNIT VIII - FACTORING

PERFORMANCE OBJECTIVES

1. Determine the GCF of two or more monomials with integral coefficients.
2. Factor a polynomial by isolating the greatest common factor.
3. Factor a polynomial that is the difference of two perfect squares.
4. Factor a trinomial that is the square of a binomial.
5. Factor a trinomial of the form $x^2 + bx + c$.
6. Factor a trinomial of the form $ax^2 + bx + c$.
7. Factor a polynomial completely.
8. Solve equations by factoring.
9. Solve word problems involving factoring.

Minimal	Average	Maximal
#1 - 8	All	All

KEY SKILLS FOR END-OF-COURSE TESTING

19. Factor polynomials completely.
20. Solve equations by factoring.
21. Solve word problems involving factoring.

UNIT VIII - FACTORING

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	145-147	307-308	183-184	290	169 305	288-289	295-297	226-227
2	151-154	312	185-186	291	169 305	288-289	295-297	226-228
3	156-159	313	188-190	307-308 317	185-187 306	292	305-307	229-232
4	160-164	313	191-192	300	181-183 309	294-295	308-310	233-236
5	167-171	315	206-211	303-306	173-177 310-311	298-299	300-305	237-241
6	172-174	318	212-214	303-306	174-180 310-311	301-302	311-313	242-245
7	175-178	319	216-219	314-315 317	186-188 306-311	304	305-310	242-245
8	180-185	320-322	221-222	491-493	189-192 434-436	310	313-316	250-252
9	186-189	324-325	223-228	---	193-196	311	313-317	250-252

UNIT VIII - FACTORING

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part II) (1977)
1	384-386	111-113	---	212-214	178-182 217
2	387-388	114-116	277-279	212-214	180-182
3	392-393	140-141	---	225-227	211
4	394-398	142-144	---	228-230	212-213
5	399-402	145-147 149-151 152-153	---	218-221	188-192
6	---	154-155	---	222-224	192-194
7	---	156-157	---	231-233	194-197
8	---	232-235	346-347	429-431	199-207
9	---	234-235	---	---	---

PERFORMANCE OBJECTIVE VIII-1

Determine the greatest common factor of two or more monomials with integral coefficients.

a) The greatest common factor of the terms of the polynomial

$24x^5 + 60x^4 - 108x^3$ is:

A. 24

B. $6x^3$

C. $12x^4$

D. $12x^3$

E. x^3

Answer ____________

State the greatest common factor of the terms of each polynomial:

b) $-30z^6 + 18z^3 - 24z^2$

Answer ____________

c) $11ab + 23a^2b^3 - 30a^3b^5$

Answer ____________

d) $75m^7n^3 \_ 60m^6n^2 - 45m^4n$

Answer ____________

PERFORMANCE OBJECTIVE VIII-2

Factors a polynomial by isolating the greatest common factor.

a) Which of the following is the prime factorization of

$50x^2y^5 - 100x^5y^2 + 125x^3y^3$?

A. $25x^2y^2(2y^3 - 4x^3 + 5xy)$

B. $25xy(2xy^4 - 4x^4y + 5x^2y^2)$

C. $25(2x^2y^5 - 4x^5y^2 + 5x^3y^3)$

D. None of the above

Answer __________

b) Determine the missing monomial factor given a polynomial factor and the product.

1) _____ $(2x^2 + 5x - 9) = 24x^5 + 60x^4 - 108x^3$.

2) _____ $(5z^4 - 3z + 4) = -30z^6 + 18z^3 - 24z^2$.

3) _____ $(11 + 23ab^2 - 30a^2b^4) = 11ab + 23a^2b^3 - 30a^3b^5$.

4) _____ $(5m^3n^2 + 4m^2n - 3) = 75m^7n^3 + 60m^6n^2 - 45m^4n$

c) Factor:

1) $9x^3 + 18x^2 + 24x$

Answer __________

2) $72mn^2 - 48mn$

Answer __________

3) $17a^3 + 9a^2 + 7a$

Answer __________

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PERFORMANCE OBJECTIVE VIII-2 (continued)

d) Factor:

1) $24a^3b + 36a^2b + 18ab$

Answer __________

2) $6x^3y^2z^2 + 14x^2y^2z + 22x^2yz^2$

Answer __________

3) $5abc^2 - 10ab^2c - 25a^2bc$

Answer __________

PERFORMANCE OBJECTIVE VIII-3

Factor a polynomial that is the difference of two perfect squares.

a) Identify the binomials that are the difference of two perfect squares by writing DS in the blank provided. (Write NOT if it is not a difference of squares.)

1) $x^2 - 9$ __________

2) $-9 + 4y^2$ __________

3) $m^6 - n^2$ __________

4) $-(x^2 + 36)$ __________

5) $6x^2 - 36$ __________

6) $-16y^2 + 1$ __________

7) $x^{2a} - 4$ __________

8) $x^4 - 16$ __________

b) Factor:

1) $x^2 - 4$

Answer __________

2) $49x^2 - 9$

Answer __________

c) Factor:

1) $-4x^2 + 1$

Answer __________

2) $9x^2 - 25y^2$

Answer __________

d) Factor:

1) $y^{2n} - 1$

Answer __________

2) $n^6 - m^6$

Answer __________

PERFORMANCE OBJECTIVE VIII-4

Factor a trinomial that is the square of a binomial.

a) Which of these is factored incorrectly?

A. $a^2 + 4a + 4 = (a + 2)(a + 2)$

B. $x^2 - 2x + 1 = (x + 1)(x + 1)$

C. $4x^2 - 12x + 9 = (2x - 3)(2x - 3)$

D. $1 + 6y + 9y^2 = (1 + 3y)(1 + 3y)$

Answer ___________

b) To be factored as the square of a binomial, the missing term of the polynomial $9x^2 + \boxed{?} + 4$ must be:

A. 36x

B. 72x

C. 12x

D. 6x

Answer ___________

c) Factor:

1) $x^2 + 2x + 1$

Answer ___________

2) $a^2 + 2ab + b^2$

Answer ___________

d) Factor:

1) $9 - 12x + 4x^2$

Answer ___________

2) $x^{2^a} + 2x^a + 1$

Answer ___________

PERFORMANCE OBJECTIVE VIII-5

Factor a trinomial of the form $x^2 + bx + c$.

a) Which of the following have a common binomial factor?

1. $x^2 + 3x - 54$
2. $x^2 - 3x - 18$
3. $x^2 + 2x - 24$

A. (a) 1 and 2

B. (b) 2 and 3

C. (c) 1 and 3

D. (d) 1, 2, and 3

E. (e) None of the above

Answer __________

b) What is the common binomial factor of the following polynomials?

$x^2 - 7x + 12$

$x^2 - 10x + 24$

$x^2 + 5x - 36$

A. $(x + 4)$

B. $(x - 4)$

C. $(x - 2)$

D. $(x - 6)$

E. None of the above

Answer __________

PERFORMANCE OBJECTIVE VIII-5 (continued)

c) Factor completely:

1) $x^2 + 10x + 24$

Answer __________

2) $x^2 + 14x + 40$

Answer __________

d) Factor completely:

1) $x^2 - 5x - 36$

Answer __________

2) $x^2 - 13x + 12$

Answer __________

PERFORMANCE OBJECTIVE VIII-6

Factor a trinomial of the form $ax^2 + bx + c$.

a) Which of the following 3 trinomials have a common binomial factor?

1. $3x^2 - 16x - 12$
2. $6x^2 - 23x - 18$
3. $6x^2 + 5x - 6$

A. 1 and 2

B. 1 and 3

C. 2 and 3

D. 1, 2, and 3

E. None of the above

Answer ____________

Factor the following trinomials:

b) $6x^2 + 19x + 10$

Answer ____________

c) $8x^2 - 34x + 21$

Answer ____________

d) $20x^2 + 7x - 6$

Answer ____________

PERFORMANCE OBJECTIVE VIII-7

Factor a polynomial completely.

Factor as completely as possible:

a) $15x^4 - 10x^3 - 25x^2$

Answer ____________

b) $a^8 - 256$

Answer ____________

c) $2y^4 - 15y^2 - 27$

Answer ____________

d) $m^4 - 13m^2 + 36$

Answer ____________

PERFORMANCE OBJECTIVE VIII-8

Solve equations by factoring.

a) The sum of the roots of the equation $x^2 + 6x + 8 = 0$ is:

A. 6

B. 8

C. -8

D. -6

Answer ____________

b) If $(x + 2)(x - 3) = 0$, which of these four conclusions follows?

A. $(x + 2) = 0$ and $(x - 3) = 0$

B. $x = 2$ and $x = -3$

C. $x^2 - x = -6$

D. Either $(x + 2) = 0$ or $(x - 3) = 0$

Answer ____________

c) Solve by factoring:

$$x^2 - 3x - 18 = 0$$

Answer ____________

d) Solve by factoring:

$$6x^2 - 15x + 6 = 0$$

Answer ____________

e) Solve by factoring:

$$4x^3 - 9x = 0$$

Answer ____________

PERFORMANCE OBJECTIVE VIII-9

Solve word problems involving factoring.

a) In a school auditorium the number of seats in each row is 3 fewer than the number of rows. How many seats are in each row if the auditorium seats 810 persons?

Answer __________

b) The Centerville Municipal Pool measures 500 feet by 40 feet. How wide must a concrete walk around the pool be if the walk is to cover an area of 1000 square feet?

Answer __________

c) Find two consecutive integers such that their product is 72 less than twice the square of the first integer.

Answer __________

d) A rectangular piece of sheet metal is 12 cm wide and 17 cm long. An equal amount is to be cut from the length and width. How much must be cut off from the length and width to leave an area of 84 cm^2?

Answer __________

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UNIT VIII - FACTORING

Answers

VIII-1

a) D

b) $-6z^2$ or $6z^2$

c) ab

d) $15m^4n$

VIII-2

a) A

b)

- 1) $12x^3$
- 2) $-6z^2$
- 3) ab
- 4) $15m^4n$

c)

- 1) $3x(3x^2 + 6x + 8)$
- 2) $24\ mn\ (3n - 2)$
- 3) $a(17a^2 + 9a + 7)$

d)

- 1) $6ab(4a^2 + 6a + 3)$
- 2) $2x^2yz(3xyz + 7y + 11z)$
- 3) $5abc(c - 2b - 5a)$

VIII-3

a)

- 1) DS
- 2) DS
- 3) DS
- 4) NOT
- 5) NOT
- 6) DS
- 7) DS
- 8) DS

b)

- 1) $(x + 2)(x - 2)$
- 2) $(7x + 3)(7x - 3)$

c)

- 1) $(1 + 2x)(1 - 2x)$
- 2) $(3x + 5y)(3x - 5y)$

d)

- 1) $(y^n + 1)(y^n - 1)$
- 2) $(n^3 + m^3)(n^3 - m^3)$

Answers (continued)

VIII-4

a) B

b) C

c)

  1) $(x + 1)^2$

  2) $(a + b)^2$

d)

  1) $(2x - 3)^2$

  2) $(x^a + 1)^2$

VIII-5

a) A

b) B

c)

  1) $(x + 6)(x + 4)$

  2) $(x + 10)(x + 4)$

d)

  1) $(x - 9)(x + 4)$

  2) $(x - 12)(x - 1)$

VIII-6

a) A

b) $(3x + 2)(2x + 5)$

c) $(4x -3)(2x -7)$

d) $(4x + 3)(5x -2)$

VIII-7

a) $5x^2 (x + 1)(3x - 5)$

b) $(a^4 + 16)(a^2 + 4)(a + 2)(a - 2)$

VIII-7 (continued)

c) $(2y^2 + 3)(y + 3)(y - 3)$

d) $(m + 2)(m - 2)(m + 3)(m - 3)$

VIII-8

a) D

b) D

c) $\{-3, 6\}$

d) $\{\frac{1}{2}, 2\}$

e) $\{-1\frac{1}{2}, 0, 1\frac{1}{2}\}$

VIII-9

a) There are 27 seats in each row

b) The walk must be 5 feet wide

c) The integers are 9 and 10 or -8 and -7

d) Five cm must be cut from the length and width so that 84 cm^2 are left.

UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

PURPOSE

In this unit, techniques for simplifying and factoring polynomial expressions are applied to algebraic fractions. Rational algebraic expressions and fractional equations are utilized in the solution of related word problems.

OVERVIEW

Simplification of algebraic fractions and the basic operations with algebraic fractions are stressed. Solutions of fractional equations and word problems involving algebraic fractions are discussed.

SUGGESTIONS TO THE TEACHER

Instructional Days: 11-14
Minimal Course Objectives: #1-7, 10
Average Course Objectives: #1-8, 10-12
Maximal Course Objectives: ALL

The parallel between the techniques for simplifying arithmetic fractions and algebraic fractions should be stressed.

VOCABULARY

algebraic fraction
complex fraction
denominator
extremes
extraneous roots
fractional equations
lowest terms
means
mixed expressions
nonrational
numerator
proportion
ratio
rational algebraic expression
reciprocal

UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

1. Reduce arithmetic fractions.
2. Multiply arithmetic fractions.
3. Divide arithmetic fractions.
4. Change mixed numbers to improper fractions.
5. Change improper fractions to mixed numbers.
6. Add and subtract fractions with like denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve equations which equal zero.

Assessment Tasks

1. Simplify each fraction.

a) $\frac{17}{34}$ a) __________

b) $\frac{24}{36}$ b) __________

c) $\frac{24}{32}$ c) __________

d) $\frac{15}{45}$ d) __________

e) $\frac{20}{32}$ e) __________

f) $\frac{16}{18}$ f) __________

g) $\frac{54}{81}$ g) __________

h) $\frac{20}{45}$ h) __________

i) $\frac{57}{95}$ i) __________

j) $\frac{36}{48}$ j) __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. a) $\frac{1}{8} \cdot \frac{1}{3}$ a) __________

b) $\frac{15}{8} \cdot \frac{1}{2}$ b) __________

c) $1\frac{3}{8} \cdot \frac{4}{5}$ c) __________

d) $3\frac{4}{5} \cdot 1\frac{2}{3}$ d) __________

3. a) $3 \div \frac{5}{8}$ a) __________

b) $\frac{7}{9} \div \frac{1}{6}$ b) __________

c) $\frac{1}{2} \div 1\frac{3}{4}$ c) __________

d) $13\frac{1}{3} \div 1\frac{1}{3}$ d) __________

4. Change to improper fractions.

a) $2\frac{8}{9}$ a) __________

b) $4\frac{2}{3}$ b) __________

c) $5\frac{5}{6}$ c) __________

d) $12\frac{3}{4}$ d) __________

5. Change to a mixed number.

a) $\frac{14}{5}$ a) __________

b) $\frac{35}{6}$ b) __________

c) $\frac{37}{3}$ c) __________

d) $\frac{95}{17}$ d) __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

6. a) $\frac{9}{10} - \frac{7}{10}$ a) ________

b) $\frac{2}{5} + \frac{4}{5}$ b) ________

c) $\frac{7}{15} - \frac{11}{15}$ c) ________

d) $\frac{19}{20} + \frac{9}{20}$ d) ________

7. a) $\frac{3}{4} - \frac{1}{2}$ a) ________

b) $\frac{3}{5} + \frac{4}{9}$ b) ________

c) $\frac{7}{12} + \frac{1}{18}$ c) ________

d) $\frac{7}{8} - \frac{3}{5}$ d) ________

8. Solve each of the following equations:

a) $x^2 + 5x - 24 = 0$ a) ________

b) $3x = 0$ b) ________

c) $x^2 - x - 6 = 0$ c) ________

d) $x(x + 3)(x - 4) = 0$ d) ________

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) $\frac{1}{2}$
   b) $\frac{2}{3}$
   c) $\frac{3}{4}$
   d) $\frac{1}{3}$
   e) $\frac{5}{8}$
   f) $\frac{8}{9}$
   g) $\frac{2}{3}$
   h) $\frac{4}{9}$
   i) $\frac{3}{5}$
   j) $\frac{3}{4}$

2. a) $\frac{1}{24}$
   b) $\frac{15}{16}$
   c) $\frac{11}{10}$
   d) $\frac{19}{3}$ or $6\frac{1}{8}$

3. a) $\frac{24}{5}$ or $4\frac{4}{5}$
   b) $\frac{14}{3}$ or $4\frac{2}{3}$
   c) $\frac{2}{7}$
   d) 10

4. a) $\frac{26}{9}$
   b) $\frac{14}{3}$
   c) $\frac{35}{6}$
   d) $\frac{51}{4}$

5. a) $2\frac{4}{5}$
   b) $5\frac{5}{6}$
   c) $12\frac{1}{3}$
   d) $5\frac{10}{17}$

6. a) $\frac{1}{5}$
   b) $\frac{6}{5}$ or $1\frac{1}{5}$
   c) $-\frac{4}{15}$
   d) $\frac{7}{5}$ or $1\frac{2}{5}$

7. a) $\frac{1}{4}$
   b) $\frac{47}{45}$ or $\frac{2}{45}$
   c) $\frac{23}{36}$
   d) $\frac{11}{40}$

8. a) $x = -8$ or 3
   b) $x = 0$
   c) $x = 3$ or -2
   d) $x = 0, -3$ or 4

UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

PERFORMANCE OBJECTIVES

1. Determine which value(s) of a variable make the denominator of a given fraction equal to zero. (II)

2. Simplify an algebraic fraction. (II)

3. Multiply algebraic fractions. (II)

4. Divide algebraic fractions. (II)

5. Simplify expressions containing both multiplication and division of algebraic fractions. (II)

6. Add (subtract) algebraic fractions with like denominators. (II)

7. Add (subtract) algebraic fractions with unlike denominators. (II)

8. Write a mixed expression as a single fraction. (II)

9. Simplify a complex algebraic fraction. (II)

10. Solve open sentences with fractional coefficients. (III)

11. Solve fractional equations. (III)

12. Solve word problems that involve the use of algebraic fractions. (III)

Minimal	Average	Maximal
#1,-7, 10	#1 - 8, 10 - 12	All

KEY SKILLS FOR END-OF-COURSE TESTING

22. Simplify an algebraic fraction.

23. Multiply and divide algebraic fractions.

24. Add and subtract algebraic fractions.

25. Solve fractional equations.

UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Forster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	197-200	347-348	183-184	200 420-421	---	362-365	331-334	370-372
2	197-200	346-348	185-186	197-203 422-425	325-330	364-370	331-334	373-375
3	205-207	353-357	188-190	204-209 426-436	325-330	371-374	340-343	376-378
4	208-210	353-357	191-192	204-209 437-442	331-333	374-378	340-343	379-381
5	211-213	353-357	---	---	---	375-377	340-343	---
6	214-217	358-360	257-258	210-213 451-456	334-340	378-380	344-347	382-383
7	214-217	360-364	259-261	218-225 461-467	346-351	381-388	348-353	384-386
8	218-219	---	---	---	---	---	353-358	---
9	210	---	---	443-446	---	389	355-358	381
10	224-225	371-374	257-258 264-265	---	356-359	---	359-363	---
11	226-227	379-383	270-271	---	356-359	394-396	359-363	388-391
12	240-245	375-378 383-397	272-288	---	360	397-405	368-373	394-397

UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part II) (1977)
1	162-163	64-66 420-422	326-328	---
2	167-169	64-66 420-425	329-331	221-226
3	190-196	67-69 426- 28	332-340	227-228
4	197-206	70-72 432-434	341-346	227-228
5	---	72	---	---
6	212-217	73-75 429-431	350-355	229
7	218-227	76-79	359-368	231-233
8	229-231	---	---	---
9	197	432-434	344-346	233 250
10	---	---	---	---
11	236-239	---	369-371	244-246 250-251
12	240-242	---	---	246-249 252-254

PERFORMANCE OBJECTIVE IX-1

Determine which value(s) of a variable make the denominator of a given fraction equal to zero.

a) Determine the values of the variable which make the denominator of the fraction $\frac{x+3}{x^2-5x-14}$ equal zero.

The sum of these values is:

A. -5

B. 0

C. 5

D. 7

E. none of the above

Answer ______________________________

Determine the values of the variables which make the denominators of the following fractions equal zero.

b) $\frac{2y^2+10y+8}{y^3-y}$ Answer ________________________

c) $\frac{x}{x^2-4}$ Answer ________________________

d) $\frac{n^2-7n}{n^2-8n+7}$ Answer ________________________

PERFORMANCE OBJECTIVE IX-2

Simplify an algebraic fraction.

a) Which of the following fractions have the same simplest form?

1. $\frac{5x^2+10x}{5x}$

2. $\frac{x^2-4}{x^2-4x+4}$

3. $\frac{x^2+5x+6}{x+3}$

A. 1 and 2

B. 1 and 3

C. 2 and 3

D. 1, 2, and 3

E. one of the above

Answer ______________

Write each of the following fractions in simplest form.

b) $\frac{y^2-3y+2}{1-y}$ Answer __________________

c) $\frac{m^2-5m+6}{m^2-9}$ Answer __________________

d) $\frac{x+3}{x^2-4x-21}$ Answer __________________

PERFORMANCE OBJECTIVE IX-3

Multiply algebraic fractions.

a) What is the simplest form of the expression

$$\frac{3x+3y}{30x} \cdot \frac{5x-5y}{x^2-y^2} \quad ?$$

A. $\frac{1}{2}$

B. $\frac{1}{2x}$

C. $\frac{x+y}{10} \cdot \frac{5}{x+y}$

D. $\frac{5}{x}$

Answer __________________

b) What is the simplest form of the expression

$$\frac{a^2-2a-8}{a^2+2a-3} \cdot \frac{a^2+3a-4}{a^2-6a+8} \cdot \frac{a^2+a-6}{a^2+6a+8} \quad ?$$

A. $\frac{a+2}{a-2}$

B. $\frac{a-1}{a+4}$

C. 1

D. $\frac{a+2}{a+3}$

Answer __________________

c) Simplify

$$\frac{a^2+2a-15}{a^2+7a+10} \cdot \frac{3a^2+12a+12}{5a^2-5a-30}.$$

Answer __________________

d) Simplify

$$\frac{p^2-4}{4p+8} \cdot \frac{4p-8}{p^2-4p+4}.$$

Answer __________________

PERFORMANCE OBJECTIVE IX-4

Divide algebraic fractions.

a) What is the simplest form of the quotient

$$\frac{x^2-16}{x+2} \div \frac{x-4}{2x+4} \text{ ?}$$

A. $\frac{2x+4}{x+2}$

B. $2x-16$

C. $\frac{x^2-2x-12}{2x-2}$

D. $2x+8$

Answer ____________

b) What is the simplest form of the quotient

$$\frac{p^2-2p-8}{p^2-p-12} \div \frac{p^2-6p+8}{p^2+p-6} \text{ ?}$$

A. 1

B. $p+2$

C. $\frac{p+2}{p-4}$

D. None of the above

Answer ____________

c) Simplify each quotient.

1. $\frac{y^2-25}{y+1} \div \frac{6y+30}{18x^2y}$

Answer ____________

2. $\frac{3a^2-48}{a^2-a-20} \div \frac{a^2-a-12}{a^2-2a-15}$

Answer ____________

d) Simplify each quotient.

1. $\frac{x}{x^2-2x+1} \div \frac{1}{1-x^2}$

Answer ____________

2. $\frac{2m^2+4m-6}{m^2-9} \div \frac{5m^2-30m-35}{m^2-10m+21}$

Answer ____________

PERFORMANCE OBJECTIVE IX-5

Simplify expressions containing both multiplication and division of algebraic fractions.

a) $\frac{x}{x + 1} \cdot \frac{6x + 18}{5x^2} \div \frac{x^2 - 9}{5x^3 + 5x^2}$

Answer __________

b) $\frac{m^2 + 2m - 15}{m^2 - m - 6} \cdot \frac{m^2 - 25}{m^2 - 4m - 5} \div \frac{m^2 + 5m + 6}{m^2 - 1}$

Answer __________

c) $\frac{10j + 5k}{j^2 - jk - 2k^2} \cdot \frac{3j - 6k}{4j^2 - k^2} \div \frac{15j - 15k}{j^2 - k^2}$

Answer __________

d) $\frac{r^2 + 11r + 18}{r^2 + 4r - 5} \quad \frac{r^2 - 7r - 8}{r^2 + 2r - 15} \quad \frac{r^2 - 6r - 7}{r^2 + 8r + 12}$

Answer __________

PERFORMANCE OBJECTIVE IX-6

Add (subtract) algebraic fractions with like denominators.

Simplify the following

a) $\frac{8}{5x} - \frac{2}{5x} + \frac{4}{5x}$

Answer ____________

b) $\frac{h^2}{h+j} - \frac{j^2}{h+j}$

Answer ____________

c) $\frac{k^2}{k-3} + \frac{9}{k-3} - \frac{6k}{k-3}$

Answer ____________

d) $\frac{4a+10}{2a^2+a-3} - \frac{2a+7}{2a^2+a-3}$

Answer ____________

PERFORMANCE OBJECTIVE IX-7

Add (subtract) algebraic fractions with unlike denominations.

a) What is the sum in simplest form of

$$\frac{3}{b^2 - 2b - 8} + \frac{2}{4 - b^2} \ ?$$

A. $\frac{1}{(b + 2)(b - 4)}$

B. $\frac{-b - 2}{-1(b - 4)(b + 2)(b - 2)}$

C. $\frac{1}{-1(b + 2)(b - 4)}$

D. $\frac{1}{(b - 2)(b - 4)}$

Answer __________

Simplify the following.

b) $\frac{3}{2m + 18} + \frac{27}{m^2 - 81}$

Answer __________

c) $\frac{5}{2z^2 - 7z - 4} - \frac{2}{2z^2 + 9z + 4}$

Answer __________

d) $\frac{2q + 3}{q - 8} - \frac{q^2 + 6q + 5}{q^2 - 7q - 8}$

Answer __________

PERFORMANCE OBJECTIVE IX-8

Write a mixed expression as a single fraction.

a) Express the following expression as a single fraction $5 + \frac{m + n}{m - n}$.

A. $\frac{5 + m + n}{m - n}$

B. $\frac{6m}{m - n}$

C. $\frac{6m - 4n}{m - n}$

D. None of the above

Answer ___________

Express each of the following expressions as a single fraction.

b) $s + 3 + \frac{1}{s + 1}$

Answer ___________

c) $e + 2f - \frac{2ef}{e-f}$

Answer ___________

d) $3d - 2 + \frac{9}{d+3}$

Answer ___________

PERFORMANCE OBJECTIVE IX-9

Simplify a complex algebraic fraction.

a) What is the simplest form of the fraction $\dfrac{\dfrac{m^2 + 4m + 4}{m^2 - m - 6}}{\dfrac{m^2 - 4}{m^2 - 5m + 6}}$?

A. $\dfrac{m + 2}{m - 3}$

B. 1

C. $\dfrac{m - 2}{m + 3}$

D. None of the above

Answer __________

Simplify each fraction.

b) $\dfrac{\dfrac{a^2 - 1}{a^2 - 6a + 5}}{\dfrac{a^2 - 4a - 5}{a^2 - 10a + 25}}$

Answer __________

c) $\dfrac{\dfrac{k - 1}{k - 2}}{\dfrac{k^2 + k - 2}{k^2 - k - 2}}$

Answer __________

d) $\dfrac{1 - \dfrac{2}{c}}{1 - \dfrac{1}{c} - \dfrac{2}{c^2}}$

Answer __________

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PERFORMANCE OBJECTIVE IX-10

Solve open sentences with fractional coefficients.

a) Solve the equations $\frac{b}{12} + \frac{b - 4}{12} = 5$ and

$\frac{4z + 3}{15} - \frac{2z - 3}{9} = \frac{3z + 2}{3} - z$

then b + z is:

A. $7\frac{1}{2}$

B. $31\frac{6}{7}$

C. 32

D. 35

E. 29

Answer __________

Solve the following equations.

b) $\frac{4x + 1}{6} + \frac{3x}{4} = \frac{2x - 4}{3}$

Answer __________

c) $\frac{b + 5}{9} - b - 3 = \frac{2b + 2}{3}$

Answer __________

d) $\frac{m + 2}{6} - \frac{m - 7}{3} = 8$

Answer __________

PERFORMANCE OBJECTIVE IX-11

Solve fractional equations.

a) If you solve each of the equations $\frac{x}{2} + \frac{x}{5} = 35$, $\frac{3}{x} + \frac{2}{5} = \frac{19}{10}$, the difference between their roots is:

A. 52

B. 48

C. 3

D. None of the above

Answer ________

Solve the following equations:

b) $\frac{5q}{q+1} - \frac{q}{q+6} = 4$

Answer ________

c) $\frac{3x-1}{x+3} - \frac{4x}{x-3} = -3$

d) $\frac{n+2}{n-2} = \frac{2}{n+2} - \frac{7}{3}$

Answer ________

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PERFORMANCE OBJECTIVE IX-12

Solve word problems that involve the use of algebraic fractions.

Solve each problem. Show all work.

a) Mr. Smithers can paint his fence in 6 hours, but it takes his daughter 9 hours to paint it. How long will it take to paint the fence if both work together?

Answer __________

b) Fred can finish his paper route in 3 hours. If his sister Jill helps him, they both can deliver the papers in 2 hours. How long would it take Ned to deliver the papers by himself?

Answer __________

c) The distance from Cooper to Asheville is 60 miles. Ms. Pawning drove from Cooper to Asheville and returned. The entire trip took 5 hours. She averaged 10 mph more while returning from Ashville. How fast did she travel each way?

Answer __________

d) A freight train travels 100 miles in the same time that a trailer truck travels 75 miles. If the rate of the train is 18 mph greater than the rate of the truck, find the rate of each.

Answer __________

UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

Answers

IX-1

a) C

b) $y \neq 0, 1, -1$

c) $x \neq 2, -2$

d) $n \neq 1, 7$

IX-2

a) B

b) $-y + 2$

c) $\frac{m - 2}{m + 3}$

d) $\frac{1}{x - 7}$

IX-3

a) B

b) C

c) $\frac{3}{5}$

d) 1

IX-4

a) D

b) C

c) 1. $\frac{3x^2y(y - 5)}{y + 1}$

2. 3

d) 1. $\frac{-x(x + 1)}{x - 1}$

2. $\frac{2(m - 1)}{5(m + 1)}$

IX-5

a) $\frac{6x}{x - 3}$

b) $\frac{m + 3}{m - 1}$

c) $\frac{1}{2j - k}$

d) $\frac{(r + 9)(r - 7)(r - 3)}{(r - 1)(r + 6)(r - 8)}$

IX-6

a) $\frac{2}{x}$

b) $h - j$

c) $k - 3$

d) $\frac{1}{a - 1}$

IX-7

a) D

b) $\frac{3}{2m - 18}$

c) $\frac{3z + 28}{(2z + 1)(z - 4)(z + 4)}$

d) $\frac{q - 2}{q - 8}$

Answers (continued)

IX-8

a) C

b) $\frac{s^2 + 4s + 4}{s + 1}$

c) $\frac{e^2 - ef - 2f^2}{e - f}$

d) $\frac{3d^2 + 7d + 3}{d + 3}$

IX-9

a) B

b) 1

c) $\frac{k + 1}{k + 2}$

d) $\frac{c}{c + 1}$

IX-10

a) D

b) $x = -2$

c) $b = -2$

d) $m = -32$

IX-11

a) B

b) $q = 24$

c) $x = -1$ or $x = 12$

d) $n = \frac{2}{5}$ or $n = -1$

IX-12

a) It will take them $3\frac{3}{5}$ hours if they work together.

b) It would take Ned 6 hours to deliver the papers himself.

c) Going to Asheville, Ms. Pawning averaged 20 mph and returning she averaged 30 mph.

d) The truck averages 54 mph and the train averages 72 mph.

UNIT X - RADICAL EXPRESSIONS

PURPOSE

This unit introduces square roots and other irrational expressions. It extends the techniques for combining similar terms to combining similar radical expressions. Proficiency in simplifying radical expressions is necessary for geometry and advanced algebra.

OVERVIEW

Conditions for writing radicals in simplest form and techniques for simplifying radical expressions are developed.

Radical expressions and extraneous roots are applied to the solution of open sentences.

SUGGESTIONS TO THE TEACHER

Instructional Days: 10-12
Minimal Course Objectives: #2 - 5, 7, 9
Average Course Objectives: #1 - 11
Maximal Course Objectives: ALL

Computation of approximate square roots could be performed by the standard square root algorithm or by the divide-and-average method.

For advanced students, the concepts of fractional exponents and higher order roots could be introduced.

Hand calculators could be a valuable aid to this unit.

VOCABULARY

conjugate
distance formula
hypotenuse
index
leg
Pythagorean Theorem
radicand
rationalize the denominator
similar radicals
square root

UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

1. Identify square factors of a whole number.
2. Identify numbers which are perfect squares.
3. State the square root of a square number.
4. Subtract directed numbers.
5. Combine similar terms.
6. Solve linear equations.
7. Multiply binomials.
8. Solve equations by factoring.

Assessment Tasks

1. a) Which of the following are square factors of 36?

   A. 2

   B. 3

   C. 4

   D. 6

   E. 9

   Answer __________

   b) Which of the following is the largest square factor of 180?

   A. 4

   B. 9

   C. 18

   D. 36

   Answer __________

   c) Name the greatest square factor of 54.

   Answer __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

1. d) Which of the following is the largest square factor of 250?

A. 5

B. 10

C. 25

D. 100

Answer __________

2. a) Which of the following is not a perfect square?

A. 1

B. 2

C. 4

D. 9

E. 121

Answer __________

b) Which of the following is a perfect square?

A. 20

B. 40

C. 50

D. 81

Answer __________

c) Which of the following is a perfect square?

A. 15

B. 21

C. 75

D. None of the above

Answer __________

d) Name the numbers between 50 and 150 which are perfect squares.

Answer __________

UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

3. a) $\sqrt{81}$ = __________ a) __________

b) $\sqrt{36}$ = __________ b) __________

c) $\sqrt{400}$ = __________ c) __________

d) $\sqrt{9}$ = __________ d) __________

4. a) $5 - 11$ a) __________

b) $-3 - (-15)$ b) __________

c) $-8 - 6$ c) __________

d) $4 - (-13)$ d) __________

5. Combine similar terms.

a) $2a + 8 - 2d + 4a - 13 - d$ a) __________

b) $4k^3 - 5k^2 - 3 + 2k^3 - 3k^2 - 6k$ b) __________

c) $3m^2 - 6m + 7 - 4m^2 - m - 9$ c) __________

d) $x - 7y + 2z - x - 11y - 8z$ d) __________

6. Solve each of the following equations:

a) $4(h + 5) + h = 35$ a) __________

b) $3(z + 1) = 2(z - 2)$ b) __________

c) $\frac{5y - 30}{7} = 0$ c) __________

d) $.07x + .04(9000 - x) = 450$ d) __________

7. a) $(3x + 2)(5x - 7)$ a) __________

b) $(7x - 2y)(3x + 8y)$ b) __________

c) $(3 - 2m)(7 - 6m)$ c) __________

d) $(3x - 7y)^2$ d) __________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

8. Solve each of the following equations:

a) $x^2 - x - 56 = 0$ a) ___________

b) $15 + x = 2x^2$ b) ___________

c) $3x^2 = 10x - 3$ c) ___________

d) $\frac{y}{3} + 1 = \frac{6}{y}$ d) ___________

UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) C, E
b) D
c) 9
d) C

2. a) B
b) D
c) D
d) 64, 81, 100, 121, 144

3. a) 9
b) 6
c) 20
d) 3

4. a) -6
b) 12
c) -14
d) 17

5. a) $6a - 3d - 5$
b) $6k^3 - 8k^2 - 6k - 3$
c) $-m^2 - 7m - 2$
d) $-18y - 6z$

6. a) $h = 3$
b) $z = -7$
c) $y = 6$
d) $x = 3000$

7. a) $15x^2 - 11x - 14$
b) $21x^2 + 50xy - 16y^2$
c) $21 - 32m + 12m^2$
d) $9x^2 - 42xy + 49y^2$

8. a) $x = 8$ or -7
b) $x = -\frac{5}{2}$ or 3
c) $x = \frac{1}{3}$ or 3
d) $x = -6$ or 3

UNIT X - RADICAL EXPRESSIONS

PERFORMANCE OBJECTIVES

1. Compute the approximate square root of a number by using a square root algorithm.

2. Write radicals in simplest form using the product and quotient properties of square roots.

3. Multiply and divide radical expressions.

4. Combine similar radicals. (II)

5. Multiply binomials containing radicals. (II)

6. Simplify a radical expression with a binomial denominator. (II)

7. Solve radical equations. (III)

8. Solve word problems involving the use of radical expressions. (III)

9. Determine the length of the unknown side of a right triangle, using the Pythagorean Theorem. (III)

10. Compute the distance between two points, using the distance formula. (II)

Minimal	Average	Maximal
#2 - 5, 7, 9	#1 - 9	ALL

KEY SKILLS FOR END-OF-COURSE TESTING

26. Write radical expressions in simplest form.

27. Solve radical equations.

UNIT X - RADICAL EXPRESSIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	387-390	408-410	338-340	273-277	---	239-241 264-265	414-417	174-176
2	398-399	418-423	332-334 342-344	260-262 269-271	398-400 402-404 405-411 415-418	243-244	396-403	407-408
3	398-400	---	344-346	263-265, 269-272	396-397 402-404	246-248	396-398	409-415
4	402-403	423-425	351-353	266-268	412-414	253	404-406	418-419
5	404-405	424	348-350	---	---	---	398	---
6	---	423-425	348-350	---	---	---	403	---
7	406-407	---	354-355	---	419-421	259-260	411-413	423-424
8	401 407-408	---	356-361	---	421-422	261	409	424
9	394-396	412-415	356-358	---	---	21,22	406-409 233-235	178-179
10	411-412	415-417	359-360	---	---	255-257	235-236	425-426

UNIT X - RADICAL EXPRESSIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objectives	Denholm et al Part II -'77	Jacobs (Part II) (1976)	Johnson et al (Part II) (1977)
1	---	176-177	---
2	368-369	157-158 162-163 166-167	274-278
3	368-369	171-173	282-285 289-290
4	370-372	168-170	281-282 286
5	373-374	---	287
6	374	---	291-294
7	375-376	---	295-297
8	376-377	---	---
9	355-356	---	267
10	---	---	268

PERFORMANCE OBJECTIVE X-1

Compute the approximate square root of a number by using a square root algorithm.

Find the following square roots correct to the nearest tenth.

a) $\sqrt{1200}$

Answer ________

b) $\sqrt{1977}$

Answer ________

c) $\sqrt{1320}$

Answer ________

d) $\sqrt{2079}$

Answer ________

PERFORMANCE OBJECTIVE X-2

Write radicals in simplest form using the product and quotient properties of square roots.

a) Which of the following expresses $4\sqrt{\frac{a^2}{8}}$ in simplest form?

A. $\frac{a}{2}$

B. $a\sqrt{2}$

C. $\frac{a\sqrt{2}}{2}$

D. None of the above

Answer ___________

b) Simplify.

1. $\sqrt{\frac{3}{4}}$

Answer ___________

2. $\sqrt{54t^7}$

Answer ___________

3. $\frac{4}{\sqrt{3}}$

Answer ___________

c) Simplify.

1. $\sqrt{20x^2y^3}$

Answer ___________

2. $\sqrt{\frac{25r^2}{12s^3}}$

Answer ___________

3. $4\sqrt{\frac{8}{49}}$

Answer ___________

PERFORMANCE OBJECTIVE X-2 (continued)

d) Simplify.

1. $\frac{a}{\sqrt{a}}$

Answer __________

2. $\frac{5}{\sqrt{24}}$

Answer __________

3. $-5\sqrt{\frac{9}{7}}$

Answer __________

PERFORMANCE OBJECTIVE X-3

Multiply and divide radical expressions.

a) Perform the indicated operation and leave answers in simplest form.

1. $2\sqrt{3} \cdot \sqrt{6}$

   Answer: ________

2. $3\sqrt{5} \cdot 5\sqrt{5}$

   Answer: ________

3. $\frac{6\sqrt{125}}{7\sqrt{5}}$

   Answer: ________

b) Perform the indicated operation and leave answers in simplest form.

1. $\sqrt{3ab^3} \cdot \sqrt{18ab}$

   Answer: ________

2. $(5\sqrt{3m})^2$

   Answer: ________

3. $\frac{12\sqrt{20}}{3\sqrt{5}}$

   Answer: ________

PERFORMANCE OBJECTIVE X-3 (continued)

c) Perform the indicated operation and leave answers in simplest form.

1. $5\sqrt{6} \cdot \frac{2}{3}\sqrt{15}$

   Answer: __________

2. $\frac{5\sqrt{24x}}{10\sqrt{6x}}$

   Answer: __________

3. $\frac{\sqrt{27} + \sqrt{75}}{\sqrt{3}}$

   Answer: __________

d) Perform the indicated operation and leave answers in simplest form.

1. $\sqrt{3xy^5} \cdot \sqrt{12x^7y^3}$

   Answer: __________

2. $\frac{5\sqrt{48a^3b}}{10\sqrt{3ab}}$

   Answer: __________

3. $\frac{6\sqrt{27} + 12\sqrt{15}}{3\sqrt{3}}$

   Answer: __________

PERFORMANCE OBJECTIVE X-4

Combine similar radicals.

a) The result of simplifying $\sqrt{4x} - \sqrt{x} - \sqrt{36x}$ is:

A. $-5\sqrt{x}$

B. $-\sqrt{33x}$

C. $-5x$

D. None of the above

Answer __________

b) Simplify.

1. $3\sqrt{5} - 5\sqrt{5} + 9\sqrt{5}$

Answer __________

2. $\sqrt{12} - \sqrt{27} + \sqrt{48}$

Answer __________

3. $\sqrt{150} - 5\sqrt{24} + 11\sqrt{54}$

Answer __________

c) Simplify.

1. $\sqrt{7} - 4\sqrt{7} + 8\sqrt{7}$

Answer __________

2. $9\sqrt{12} + 3\sqrt{48} - 7\sqrt{27}$

Answer __________

3. $\sqrt{\frac{2}{3}} + 5\sqrt{\frac{1}{6}} - 3\sqrt{\frac{3}{2}}$

Answer __________

PERFORMANCE OBJECTIVE X-4 (continued)

d) Simplify.

1. $7\sqrt{2} - 7\sqrt{3} + 4\sqrt{3} - \sqrt{2}$

Answer __________

2. $3\sqrt{x} + 5\sqrt{y} - 7\sqrt{x} - 5\sqrt{y}$

Answer ________

3. $\frac{1}{3}\sqrt{\frac{2}{27}} - \frac{1}{6}\sqrt{\frac{1}{3}}$

Answer __________

PERFORMANCE OBJECTIVE X-5

Multiply binomials containing radicals.

a) Which of the following pairs of expressions are conjugates?

A. $(2 + \sqrt{5})\ (2 + \sqrt{5})$

B. $(2 - \sqrt{5})\ (2 + \sqrt{5})$

C. $(3 + \sqrt{7})\ (3 - \sqrt{7})$

D. $(\sqrt{2} + \sqrt{5})\ (\sqrt{2} + \sqrt{5})$

Answer __________

b) Express each in simplest form.

1. $(3 + \sqrt{5})(3 - \sqrt{5}) =$ __________
2. $(1 + \sqrt{6})(2 - \sqrt{6}) =$ __________
3. $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) =$ __________
4. $(2\sqrt{3} + 1)^2 =$ __________

c) Evaluate $x^2 + 2x - 1$ for

$x = (\sqrt{2} - 1)$

Answer __________

d) Simplify.

1. $(4\sqrt{3} - 3\sqrt{5})(3\sqrt{3} + 4\sqrt{5}) =$

____

2. $(2\sqrt{3} + \sqrt{2})^2 =$ __________

PERFORMANCE OBJECTIVE X-6

Simplify a radical expression with a binomial denominator.

a) To write $\frac{1}{\sqrt{7}-2}$ in simplest form:

A. Multiply the denominator by $\sqrt{7}$

B. Multiply by $\frac{\sqrt{7}}{\sqrt{7}}$

C. Multiply by $\frac{\sqrt{7}+2}{\sqrt{7}+2}$

D. Already in simplest form

Answer ________

b) The simplest form of $\frac{\sqrt{3}}{\sqrt{3}+6}$ is:

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{-1+2\sqrt{3}}{11}$

D. $\frac{\sqrt{6}}{-11}$

Answer ________

c) Simplify $\frac{9\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$

Answer ________

d) Simplify $\frac{1}{3\sqrt{5}+4}$

Answer ________

PERFORMANCE OBJECTIVE X-7

Solve radical equations.

a) Find the solution to the equation $\sqrt{1} + 2x = \sqrt{5}$.

A. 1

B. $\frac{\sqrt{5} - 1}{2}$

C. $\frac{\sqrt{5}}{2}$

D. None of the above

Answer ___________

b) Find the solution to the equation $\sqrt{7 + 3x} = -4$

A. 3

B. -3

C. ϕ

D. None of the above

Answer ___________

c) Solve the following equations.

1. $\sqrt{5x} = 15$

Answer ___________

2. $\sqrt{3a} = 8$

Answer ___________

3. $8 = \sqrt{x - 9}$

d) Solve the following equations.

1. $\sqrt{t} = t - 6$

Answer ___________

2. $2\sqrt{x} - x = -3$

Answer ___________

PERFORMANCE OBJECTIVE X-8

Solve word problems involving the use of radical expressions.

Solve each word problem. Show all work.

a) Twice the square root of a number is 44. Find the number.

Answer __________

b) When 9 is added to a number, the square root is 10. Find the number.

Answer __________

c) If an object is dropped from a certain height, the time it takes to fall can be found by the formula $t = \sqrt{\frac{s}{16}}$, where t is time and s is the distance in feet. Find the number of seconds it takes a baseball that is hit to a height of 200 feet to fall to the ground from its highest point.

Answer __________

d) Heron's (Hero's) formula, $A = \sqrt{s(s-a)(s-b)(s-c)}$, is used to find the area of a triangle when only the lenths of the sides (a, b, and c) are known and s is the value of one-half the perimeter. Find the area of a triangle whose sides are 12 cm, 16 cm, and 20 cm (use a square root table).

Answer __________

PERFORMANCE OBJECTIVE X-9

Determine the length of the unknown side of a right triangle using the Pythagorean Theorem.

Solve each problem.

a) Find the hypotenuse of a right triangle if the lengths of the shorter sides are 18 cm and 24 cm.

Answer __________

b) A television antenna tower is 350 m tall. If a cable 500 m long were used to support the tower 300 m from the ground, how far from the base of the tower will the cable be anchored to the ground?

Answer __________

c) Find the length of the diagonal of a rectangle that is 9 inches long and 4 inches wide.

Answer __________

d) Romeo placed the bottom of his ladder 8 feet from the wall of Juliet's house. The top of the ladder just reached the bottom of Juliet's window which was 15 feet above the ground. How long was Romeo's ladder?

Answer __________

PERFORMANCE OBJECTIVE X-10

Compute the distance between two points using the distance formula.

a) Find the distance between (-3, 1) and (5, 4).

Answer __________

b) Find the distance between (3, 3) and (9, -5).

Answer __________

c) The vertices of a triangle are (0, 0), (3, 4), (8,0). Find its perimeter.

Answer __________

d) The vertices of a diamond are:

A. (-5, 1)

B. (-2, 5)

C. (1, 1)

D. (-2, -3).

Find its perimeter.

Answer __________

UNIT X - RADICAL EXPRESSIONS

Answers

X-1

a) 34.6

b) 44.5

c) 36.3

d) 45.6

X-2

a) B

b) 1. $\frac{1}{2}\sqrt{3}$

2. $3t^3\sqrt{6t}$

3. $\frac{4\sqrt{3}}{3}$

c) 1. $2xy\sqrt{5y}$

2. $\frac{5r\sqrt{3s}}{6s^2}$

3. $\frac{8\sqrt{2}}{7}$

d) 1. $\sqrt{a}$

2. $\frac{5\sqrt{6}}{2}$

3. $\frac{-15\sqrt{7}}{7}$

X-3

a) 1. $6\sqrt{2}$

2. 75

3. $\frac{30}{7}$

b) 1. $3ab^2\sqrt{6}$

2. 75 m

3. 8

X-3 (continued)

c) 1. $10\sqrt{10}$

2. 1

3. 8

d) 1. $6x^4y^4$

2. 2a

3. $6 + 4\sqrt{5}$

X-4

a) A

b) 1. $7\sqrt{5}$

2. $3\sqrt{3}$

3. $28\sqrt{6}$

c) 1. $5\sqrt{7}$

2. $9\sqrt{3}$

3. $-\frac{1}{3}\sqrt{6}$

d) 1. $6\sqrt{2} - 3\sqrt{3}$

2. $-4\sqrt{x}$

3. $\frac{2\sqrt{6} - 3\sqrt{3}}{54}$

X-5

a) B, C.

b) 1. 4

2. $-4 + \sqrt{6}$

Answers (continued)

X-5 (continued)

3. -1

4. $13 + 4\sqrt{3}$

c) 0

d) 1. $-24 + 7\sqrt{15}$

2. $14 + 4\sqrt{6}$

X-6

a) C

b) C

c) $7 - \sqrt{15}$

d) $\frac{3\sqrt{5} - 4}{29}$

X-7

a) B

b) C

c) 1. $x = 45$

2. $a = 21\frac{1}{3}$

3. $73 = x$

d) 1. $\{9\}$

2. $\{9\}$

X-8

a) $x = 484$

b) $x = 91$

c) $t = \frac{5\sqrt{2}}{2}$ sec

d) $A = 96\ \text{cm}^2$

X-9

a) 30 cm

b) 400 m

c) $\sqrt{97}$ inches

d) 17 feet

X-10

a) $d = \sqrt{73}$

b) $d = 10$

c) Perimeter $= 13 + \sqrt{41}$

d) Perimeter $= 20$

UNIT XI - QUADRATIC EQUATIONS

PURPOSE

For students who will continue their study of mathematics in geometry and advanced algebra, a knowledge of the general solution for any quadratic equation is important. The main emphasis of this unit is placed on the quadratic formula and its application.

OVERVIEW

Three methods for solving a quadratic equation, graphing, completing the square, and the quadratic formula are presented. (The factoring method was introduced in Unit VIII.) Quadratic functions and their graphs are also discussed. As a culminating activity for this unit, students solve word problems involving quadratic relations.

SUGGESTIONS TO THE TEACHER

Instructional Days: 8-10
Minimal Course Objectives: #1, 2, 3
Average Course Objectives: #1, 2, 3, 5
Maximal Course Objectives: ALL

Familiarization with the quadratic formula is a minimal outcome of this unit. It is strongly suggested that advanced students learn the derivation of the quadratic formula.

VOCABULARY

- completing the square
- discriminant
- nature of the roots of a quadratic equation
- parabola
- quadratic equation
- quadratic formula
- quadratic relation
- symmetry
- vertex

UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

1. Add fractions with unlike denominators.
2. Multiply whole numbers and fractions by $\frac{1}{2}$.
3. Square arithmetic fractions.
4. Square a binomial.
5. Factor a trinomial square.
6. Simplify square roots.
7. Evaluate algebraic expressions containing square roots.
8. Solve equations by factoring.
9. Graph a quadratic function.

Assessment Tasks

1. a) $\frac{3}{8} + \frac{3}{4}$ a) ____________

   b) $\frac{1}{2} + \frac{1}{3}$ b) ____________

   c) $\frac{4}{5} + \frac{2}{3}$ c) ____________

   d) $\frac{1}{6} + \frac{3}{4}$ d) ____________

   e) $\frac{13}{36} + \frac{3}{4}$ e) ____________

   f) $\frac{19}{25} + \frac{3}{10}$ f) ____________

   g) $\frac{8}{49} + \frac{9}{14}$ g) ____________

   h) $3 + \frac{1}{2}$ h) ____________

   i) $5 + \frac{1}{9}$ i) ____________

   j) $-1 + \frac{25}{4}$ j) ____________

2. a) $\frac{1}{2} \cdot \frac{3}{4}$ a) ____________

   b) $\frac{1}{2} \cdot 8$ b) ____________

   c) $\frac{1}{2} \cdot 11$ c) ____________

   d) $15 \cdot \frac{1}{2}$ d) ____________

   e) $24 \cdot \frac{1}{2}$ e) ____________

   f) $\frac{5}{6} \cdot \frac{1}{2}$ f) ____________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. g) $\frac{9}{17} \cdot \frac{1}{2}$ g) ______

h) $\frac{1}{2} \cdot \frac{4}{5}$ h) ______

i) $12 \cdot \frac{1}{2}$ i) ______

j) $\frac{1}{2} \cdot \frac{16}{25}$ j) ______

3. a) $\left(-\frac{2}{3}\right)^2$ a) ______

b) $\left(\frac{3}{4}\right)^2$ b) ______

c) $\left(\frac{5}{6}\right)^2$ c) ______

d) $\left(1\ \frac{1}{2}\right)^2$ d) ______

e) $\left(-\ \frac{3}{4}\right)^2$ e) ______

f) $\left(2\ \frac{2}{3}\right)^2$ f) ______

g) $\left(-\frac{16}{7}\right)^2$ g) ______

h) $\left(1\ \frac{2}{5}\right)^2$ h) ______

i) $\left(\frac{17}{5}\right)^2$ i) ______

4. a) $(2x - 3)^2$ a) ______

b) $(2x + 1)^2$ b) ______

c) $(5x - 4)^2$ c) ______

d) $(\frac{1}{2}x + 5)^2$ d) ______

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

5. a) $25x^2 + 30x + 9$ a) ________

b) $49a^2 - 28ab + 4b^2$ b) ________

c) $9x^2 - 42xy + 49y^2$ c) ________

d) $25x^2 - 10x + 1$ d) ________

6. Simplify each of the following expressions.

a) $\sqrt{\frac{(2x + 3)^2}{16}}$ a) ________

b) $\sqrt{81(x - 25)^2}$ b) ________

c) $\frac{6 + \sqrt{52}}{2}$ c) ________

d) $\frac{-26 + \sqrt{144}}{4}$ d) ________

7. If $a = 3$, $b = -2$, $c = -1$, and $d = 12$, find the value of each of the following expressions:

a) $\sqrt{\frac{-2a(-d)}{bc}}$ a) ________

b) $\sqrt{\frac{-9}{cd}}$ b) ________

c) $\sqrt{\frac{8abc}{d}}$ c) ________

d) $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ d) ________

8. Solve each of the following equations:

a) $a^2 - 8a = 20$ a) ________

b) $2h^2 = 210 + 16h$ b) ________

c) $x = \frac{40}{x - 3}$ c) ________

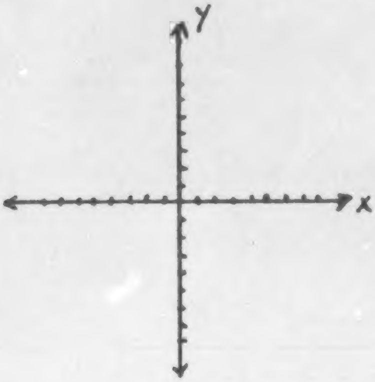
d) $\frac{x}{3} = \frac{8}{x + 2}$ d) ________

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

9. Graph each of the following functions:

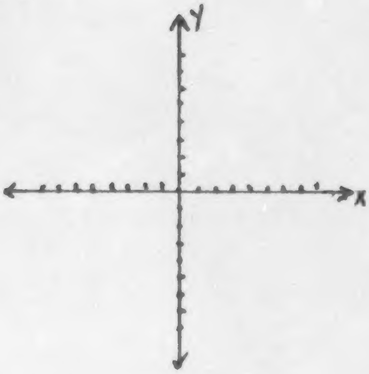
a) $y = x^2 + 1$



a)

x	y
0	
1	
2	
3	
-1	
-2	
-3	

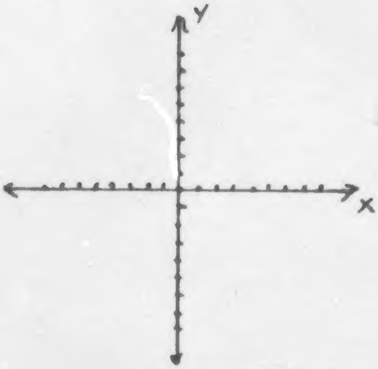
b) $y = -2x^2 + 7$



b)

x	y
0	
1	
2	
3	
-1	
-2	
-3	

c) $y = \frac{1}{2} x^2 - 3$



c)

x	y
0	
1	
2	
3	
-1	
-2	
-3	

UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) $\frac{9}{8}$ or $1\frac{1}{8}$
   b) $\frac{5}{6}$
   c) $\frac{22}{15}$ or $1\frac{7}{15}$
   d) $\frac{11}{12}$
   e) $\frac{10}{9}$ or $1\frac{1}{9}$
   f) $\frac{53}{50}$ or $1\frac{3}{50}$
   g) $\frac{79}{98}$
   h) $3\frac{1}{2}$
   i) $5\frac{1}{9}$
   j) $\frac{21}{4}$ or $5\frac{1}{4}$

2. a) $\frac{3}{8}$
   b) 4
   c) $\frac{11}{2}$ or $5\frac{1}{2}$
   d) $\frac{15}{2}$ or $7\frac{1}{2}$
   e) 12
   f) $\frac{5}{12}$
   g) $\frac{9}{34}$
   h) $\frac{2}{5}$
   i) 6
   j) $\frac{8}{25}$

3. a) $\frac{4}{9}$
   b) $\frac{9}{16}$
   c) $\frac{25}{36}$
   d) $\frac{9}{4}$
   e) $\frac{9}{16}$
   f) $\frac{64}{9}$
   g) $\frac{256}{49}$
   h) $\frac{49}{25}$
   i) $\frac{289}{25}$

4. a) $4x^2 - 12x + 9$
   b) $4x^2 + 4x + 1$
   c) $25x^2 - 40x + 16$
   d) $\frac{1}{4}x^2 + \frac{25}{4}x + 25$

5. a) $(5x + 3)^2$
   b) $(7a - 2b)^2$
   c) $(3x - 7y)^2$
   d) $(5x - 1)^2$

6. a) $\frac{2x + 3}{4}$
   b) $9(x - 25)$
   c) $3 + \sqrt{13}$
   d) $-\frac{7}{2}$ or $-3\frac{1}{2}$

ENTERING PERFORMANCE OBJECTIVES

Answers (continued)

7. a) 6

   b) $\frac{1}{2}$

   c) 2

   d) 1

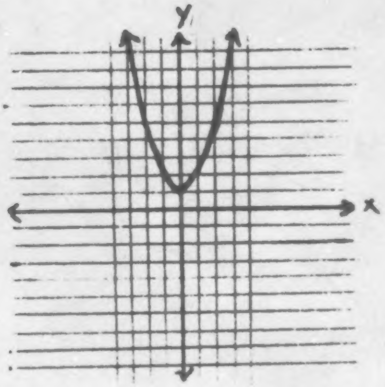
8. a) a = 10 or -2

   b) h = 15 or -7

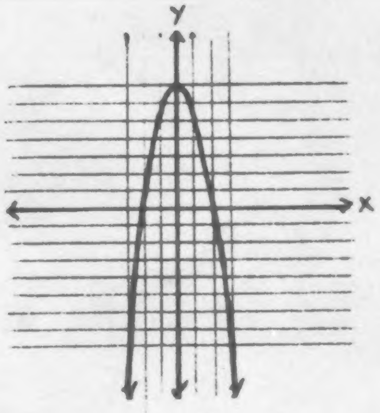
   c) x = 8 or -5

   d) x = -6 or 4

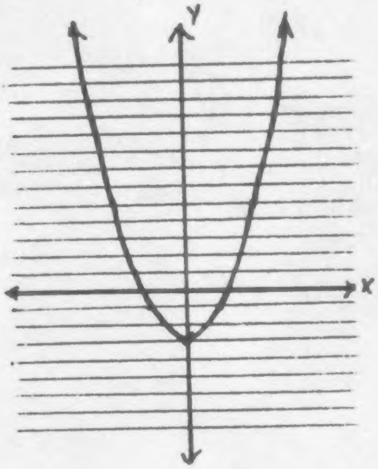
9. a)



b)



c)



UNIT XI - QUADRATIC EQUATIONS

PERFORMANCE OBJECTIVES

1. Solve quadratic equations by applying the square root property of equality.(III)
2. Solve quadratic equations by completing the square. (III)
3. Solve quadratic equations using the quadratic formula. (II)
4. Solve quadratic functions graphically. (III)
5. Solve word problems that involve the quadratic formula. (III)

Minimal	Average	Maximal
#1-3	#1-3, 5	ALL

KEY SKILLS FOR END-OF-COURSE TESTING

28. Solve a quadratic equation by completing the square or applying the quadratic formula.

UNIT XI - QUADRATIC EQUATIONS

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	391-392 432-434	---	---	494-498	430-433 438-439	337-339	---	434-436
2	435-437	431	375-377	499-504	440-442	340-345	449-452	437-439
3	438-441	434-436	379-381	504-508	443-447	346-348	452-455	440-443
4	440	440-444	366-374	487-490	461-464	330-331	456-459	488-490
5	442-445	437-438	385-387	---	450-453 458-460	349-352	463-467	448-455

Objective	Denholm et al Part II -'77	Jacobs (Part II) (1976)	Johnson et al (Part II) (1977)
1	378-382	432-434	365-366, 371
2	383-385	435-437	366-369
3	386-388	438-440	372-376
4	385	398-417	389-393
5	119-122	444-446	377-385

PERFORMANCE OBJECTIVE XI-1

Solve equations by applying the square root property of equality.

Solve the following equations by applying the square root property of equality.

a) 1. $5x^2 = 500$ Answer: ________

2. $(x - 3)^2 = 49$ Answer: ________

3. $x^2 + 10x + 25 = 121$ Answer: ________

b) 1. $2x^2 - 3 = 0$ Answer: ________

2. $(x + \frac{1}{3})^2 = \frac{25}{49}$ Answer: ________

3. $9x^2 - 6x + 1 = 16$ Answer: ________

c) 1. $z^2 + .01 = .37$ Answer: ________

2. $(y - 9)^2 = 64$ Answer: ________

3. $(z - 4)^2 = 11$ Answer: ________

d) 1. $7x^2 = 4x^2 + .75$ Answer: ________

2. $(x + 5)^2 = 44$ Answer: ________

3. $4x^2 - 12x + 9 = 169$ Answer: ________

PERFORMANCE OBJECTIVE XI-2

Solve quadratic equations by completing the square.

Solve the following equations by completing the square.

a) $x^2 + 4x - 12 = 0$

Answer ___________

b) $x^2 - 4x + 2 = 0$

Answer ___________

c) $2x^2 = -7x - 3$

Answer ___________

d) $3x^2 - 6x = 2$

Answer ___________

PERFORMANCE OBJECTIVE XI-3

Solve a quadratic equation using the quadratic formula.

Use the quadratic formula to solve the following equations.

a) $2x^2 - 7 = 0$

Answer __________

b) $x^2 = 3 - 3x$

Answer __________

c) $2x^2 + 5x = -2$

Answer __________

d) $8x + 1 = -3x^2$

Answer __________

PERFORMANCE OBJECTIVE XI-4

Solve quadratic functions graphically.

a) Solve graphically $x^2 - 4 = y$ by making a table of values.

x	y
-2	
-1	
0	
1	
2	

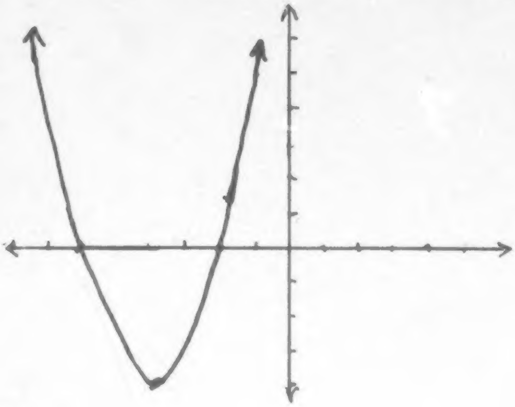
b) Solve graphically $x^2 - 6x + 9 = y$ by making a table of values.

x	y
1	
2	
3	
4	
5	

c) Solve graphically $(x + 2)^2 - 1 = y$ by making a table of values.

x	y
0	
-1	
-2	
-3	
-4	

d) Given the graph of $y = x^2 + 8x + 12$, find the values for x that would make $y = 0$.



Answer: x = _____

x = _____

PERFORMANCE OBJECTIVE XI-5

Solve word problems that involve the quadratic formula.

a) The area of the Student Government Association bulletin board is 24 square feet. If it is 2 feet longer than it is wide, find the dimensions.

Answer __________

b) In an apartment building there are 8 fewer apartments per floor than there are floors. If the building has 609 units, how many floors are in the building?

Answer __________

c) In computing the total cost, c, of setting up a factory to manufacture mag wheels, A.J. came up with the formula $C = 500 + 10x + x^2$, where x is the number of wheels produced. How many wheels can be produced at an initial cost of $3500.00?

Answer __________.

d) The Vera City School District has instituted a voucher system under which each student goes to the school of his choice. The city pays the school $(36 + 4x)$ dollars per student per month. How many students, x, would a school have to have on its rolls to be paid $1600 per month?

Answer __________

UNIT XI - QUADRATIC EQUATIONS

Answers

1. a) 1. $x = \pm 10$

2. $x = 10$ or $x = -4$

3. $x = 6$ or $x = -16$

b) 1. $x = \pm \frac{\sqrt{6}}{2}$

2. $x = \frac{22}{21}$ or $x = -\frac{8}{21}$

3. $x = \frac{5}{3}$ or $x = -\frac{2}{3}$

c) 1. $z = \pm .6$

2. $y = 17$ or $y = 1$

3. $z = 4 \pm \sqrt{11}$

d) 1. $x = \pm .5$

2. $x = -5 \pm 2\sqrt{11}$

3. $x = 8$ or $x = -5$

2. a) $x = -6$ or $x = 2$

b) $x = 2 \pm \sqrt{2}$

c) $x = -3$ or $x = -\frac{1}{2}$

d) $x = \frac{3 \pm \sqrt{15}}{3}$

3. a) $x = \pm \frac{\sqrt{14}}{2}$

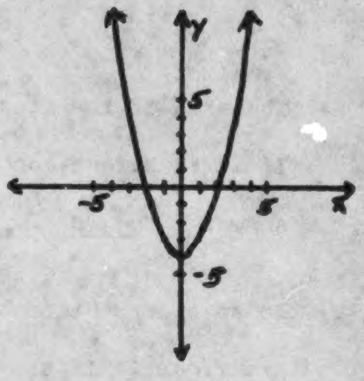
b) $x = \frac{-3 \pm \sqrt{21}}{2}$

c) $x = -2, -\frac{1}{2}$

d) $x = \frac{-4 \pm \sqrt{13}}{3}$

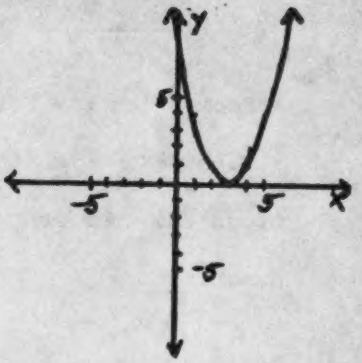
4. a)

x	y
-2	0
-1	-3
0	-4
1	-3
2	0



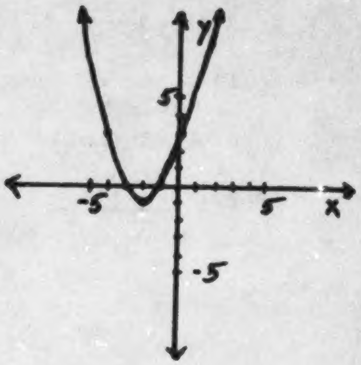
b)

x	y
1	4
2	1
3	0
4	1
5	4



c)

x	y
0	3
-1	0
-2	-1
-3	0
-4	3



5. a) $x = 4$

b) $x = 29$

c) $x = 50$

d) $x = 16$

ENRICHMENT

UNIT XII - NUMERICAL TRIGONOMETRY

PURPOSE

Familiarity with the basic trigonometric ratios and right triangles will be a valuable aid to the student who will continue his/her mathematical studies in geometry and trigonometry. This unit will provide a background in the rudiments of right triangle trigonometry. The ideas discussed are simple enough to enable slower classes to grasp the basic concepts, if desired.

OVERVIEW

Students are introduced to the basic terminology of the right triangle. Emphasis is placed on the sine, cosine, and tangent ratios and their relationships to the sides of a right triangle. Students are expected to use these ratios and a table of trigonometric values to find the missing parts of a right triangle.

SUGGESTIONS TO THE TEACHER

Instructional Days: 6-8
Minimal Course Objectives: None
Average Course Objectives: 1, 5, and 8
Maximal Course Objectives: ALL

Students will find it especially handy to memorize the values of sine, cosine, and tangent of 0°, 30°, 45°, 60°, and 90°. Hand calculators will greatly simplify the computation of this unit.

VOCABULARY

acute angle
adjacent leg
angle of depression
angle of elevation
cosine
hypotenuse
legs of a right triangle
obtuse angle
opposite leg
Pythagorean Theorem
right angle
right triangle
similar triangles
sine
tangent
trigonometric ratio
trigonometry

ENRICHMENT

UNIT XII - NUMERICAL TRIGONOMETRY

PERFORMANCE OBJECTIVES

1. State the sine, cosine, and tangent ratios in terms of the sides of a right triangle. (I)

2. Determine the value of the sine, cosine, or tangent of a given angle of a right triangle when given the lengths of the sides of the triangle. (II)

3. Locate in a table of trigonometric values the sine, cosine, or tangent of a given angle. (I)

4. Determine the measure of an angle from a table of trigonometric values when given the value of its sine, cosine, or tangent. (I)

5. Determine the measure of a given angle of a right triangle from a table of trigonometric values when given the lengths of any two sides of the triangle. (II)

6. State from memory the values of the following:

   $\sin 0^\circ$ $\quad$ $\sin 30^\circ$ $\quad$ $\sin 90^\circ$

   $\cos 0^\circ$ $\quad$ $\cos 60^\circ$ $\quad$ $\cos 90^\circ$

   $\tan 0^\circ$ $\quad$ $\tan 45^\circ$ $\quad$ $\tan 90^\circ$ (I)

7. Demonstrate that the following statements are true.

   $\cos A \tan A = \sin A$ $\quad$ $\tan A = \frac{\sin A}{\cos A}$

   $\cos A = \frac{\sin A}{\tan A}$ $\quad$ $\sin^2 A + \cos^2 A = 1$

8. Determine the length of a specified side of a right triangle when given the length of another side and the measure of one of the acute angles. (III)

9. Solve word problems involving right triangles, utilizing the sine, cosine, or tangent ratios. (III)

Minimal	Average	Maximal
None	#1, 5, 8	All

UNIT XII - NUMERICAL TRIGONOMETRY

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	464-465	462-465	463-465	517,521	514-520	419-424	517-521	516-518
2	464-466	---	463-465	517-520 521-524	514-520	421,426	517-521	516-518
3	467-468	461-462	467-468	517-520 521-524	521-522	419-426	523-524	519-521
4	467-468	461-462	467-468	518,520 522-524	521-522	422,426	523-524	519-521
5	---	462-465	469-471	517-520 525-529	524-527	421,426	---	---
6	467	---	467-468	---	---	---	529-532	---
7	---	---	---	---	---	---	522	---
8	469-470	462-467	469-471	517-520 525-529	524-527	422	523-527	520
9	471-472	468-469	473-475	529	524-527	418,422 425-431	523-528	523-526

UNIT XII - NUMERICAL TRIGONOMETRY

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Denholm et al Part II -'77	Jacobs (Part II) (1976)	Johnson et al (Part II) (1977)
1	430-440	455-457 463-466	357
2	433 436 439	457-459 463-465	357
3	431,436 438	457-465	---
4	431	457-465	---
5	433 436 439	457-459 463-466	---
6	---	---	---
7	---	---	---
8	433 437 440	460-462 467-470	359
9	---	---	359-361

PERFORMANCE OBJECTIVE XII-1

State the sine, cosine, and tangent ratios in terms of the sides of a right triangle.

a) The sine of an angle of a right triangle is the ratio of:

A. $\frac{\text{side adjacent to the angle}}{\text{hypotenuse}}$

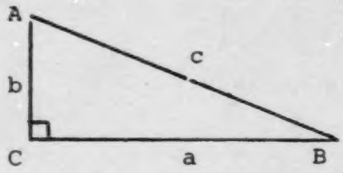
B. $\frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}$

C. $\frac{\text{side adjacent to the angle}}{\text{side opposite the angle}}$

D. $\frac{\text{side opposite the angle}}{\text{hypotenuse}}$

Answer __________

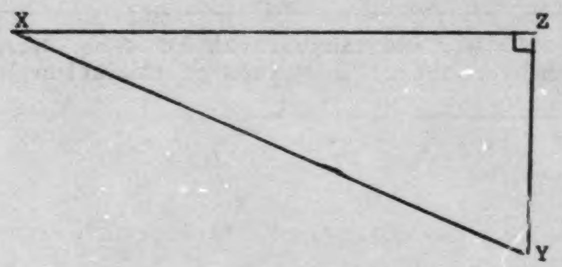
b)



sin A = __________

cos B = __________

tan B = __________

c) 

tan Y = __________

A. $\frac{XY}{YZ}$

B. $\frac{YZ}{XZ}$

C. $\frac{XZ}{XY}$

D. $\frac{XZ}{YZ}$

E. None of the above

Answer __________

d) State the following in terms of: 1) the side opposite the angle,
2) the side adjacent to the angle, and
3) the hypotenuse, of a right triangle.

1) sine of an angle: __________

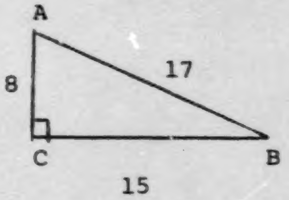
2) cosine of an angle __________

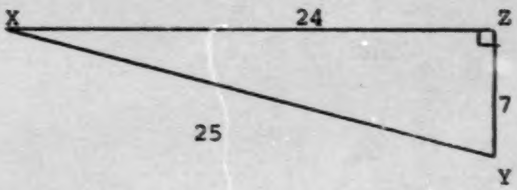
3) tangent of an angle __________

PERFORMANCE OBJECTIVE XII-2

Determine the value of the sine, cosine, and tangentsratios of an angle of a right triangle when given the lengths of the sides of the triangle.

Refer to the drawings below to complete the following: (Write each answer as a decimal correct to the nearest hundredth.)
a decimal correct to the nearest hundredth





a) sin A = _____

cos A = _____

tan A = _____

b) sin B = _____

cos B = _____

tan B = _____

c) sin X = _____

cos X = _____

tan X = _____

d) sin Y = _____

cos Y = _____

tan Y = _____

PERFORMANCE OBJECTIVE XII-3

Locate in a table of trigonometric values the sine, cosine, or tangent of a given angle.

Use a table of trigonometric values to find the following:

a) sin 15° = _____

cos 79° = _____

tan 80° = _____

b) sin 78° = _____

cos 23° = _____

tan 41° = _____

c) sin 64° = _____

cos 89° = _____

tan 13° = _____

d) 5 sin 42° = _____

3.5 cos 13° = _____

9 tan 39° = _____

PERFORMANCE OBJECTIVE XII-4

Determine the measure of an angle from a table of trigonometric values when given the value of its sine, cosine, or tangent.

State the following to the nearest degree:

a) If sin θ = .3090, then θ = _____

b) If cos θ = .4848, then θ = _____

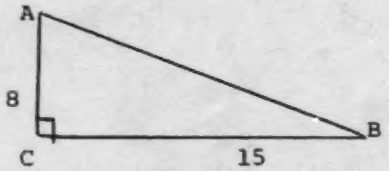
c) If tan θ = 2.0503, then θ = _____

d) If cos θ = .8310, then θ = _____

PERFORMANCE OBJECTIVE XII-5

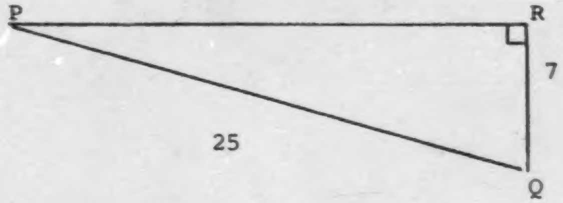
Determine the measure of a given angle of a right triangle from a table of trigonometric values when given the lengths of any two sides of the triangle.

a)



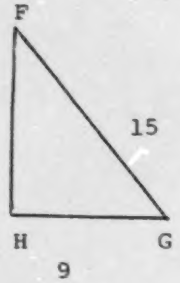
Find the measure of ∢A to the nearest degree. __________

b)



Find the measure of ∢Q to the nearest degree. _____

c)



Find the measure of ∢F to the nearest degree. _____

d) In Δ ABC with right angle C, AB = 13" and BC = 5". Find the measure of ∢A to the nearest degree.

PERFORMANCE OBJECTIVE XII-6

State from memory the values of the following:

sin 0°	sin 30 °	sin 90°
cos 0°	cos 60°	cos 90°
tan 0°	tan 45°	tan 90°

a) sin 0° = ?

A. 0

B. cos 90°

C. tan 0°

D. All of the above

E. None of the above

Answer ____________

b) sin 30° < ?

A. cos 60°

B. tan 45°

C. cos 90°

D. All of the above

E. None of the above

Answer ____________

c) Complete the following:

1) cos 0° ______

2) tan 0° ______

3) cos 60° ______

d) Complete the following:

1) tan 45° = ______

2) sin 90° = ______

3) sin 30° = ______

PERFORMANCE OBJECTIVE XII-7

Demonstrate that the following statements are true:

$\cos A \tan A = \sin A$ $\sin^2 A + \cos^2 A = 1$

$\frac{\sin A}{\cos A} = \tan A$ $\frac{\sin A}{\tan A} = \cos A$

a) Show that $\cos A \tan A = \sin A$

b) Show that $\frac{\sin A}{\cos A} = \tan A$

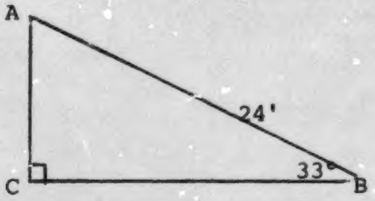
c) Show that $\frac{\sin A}{\tan A} = \cos A$

d) Show that $\sin^2 A + \cos^2 A = 1$

PERFORMANCE OBJECTIVE XII-8

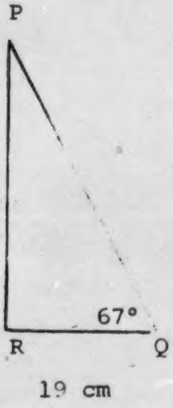
Determine the length of a specified side of a right triangle given the length of another side and the measure of one of the acute angles.

a)



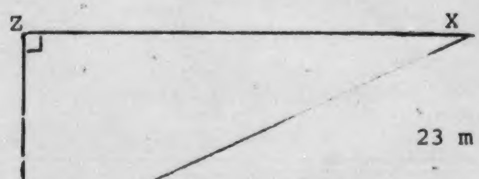
Find AC. ______

b)



Find PR. ______

c)



Find YZ. ______

d) In ΔABC, m $\measuredangle C = 90°$, m $\measuredangle A = 27°$ and BC = 10.19". Find AC. ______

PERFORMANCE OBJECTIVE XII-9

Solve word problems involving right triangles utilizing the sine, cosine, or tangent ratios.

a) Dr. Doobee, a ham radio operator, builds an antenna 60 feet tall for his radio. He mounts a support cable between the ground and the top of the antenna. If the support cable makes an angle of 70° with the ground, how long is the cable? How far from the base of the antenna is the cable anchored in the ground? (Find answers to the nearest tenth.)

b) Franklin Benjamin is flying a kite at the end of a 300' long string. If the kite is flying at a height of 200', what angle (to the nearest degree) does the string make with the ground?

c) At the end of his act, the Great Mitch Donkowski, a circus tightrope walker, walks down a cable from a platform 50 feet high to the ground. If the cable makes an angle of 25° with the ground, how long is the cable? (Find answer correct to the nearest tenth.)

d) The rope tow at Mrs. Gurr's Ski Lodge makes a 42° angle with the horizontal. If the rope tow is 3000 feet long, how many feet higher is the top of the slope than the bottom of the rope tow?

Answers

1. a) D

   b) $\sin A = \frac{a}{c}$

   $\cos B = \frac{a}{c}$

   $\tan B = \frac{b}{a}$

   c) D

   d) 1) sine of an angle = $\frac{\text{side opposite the angle}}{\text{hypotenuse}}$

   2) cosine of an angle = $\frac{\text{side adjacent to the angle}}{\text{hypotenuse}}$

   3) tangent of an angle = $\frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}$

2. a) sin A = .88

   cos A = .47

   tan A = 1.88

   b) sin B = .47

   cos B = .88

   tan B = .53

   c) sin X = .28

   cos X = .96

   tan X = .29

   d) sin Y = .96

   cos Y = .28

   tan Y = 3.43

3. a) sin 15° = .2588

   cos 79° = .1908

   tan 80° = 5.6713

Answers (continued)

3. b) $\sin 78° = .9781$

$\cos 23° = .9205$

$\tan 41° = .8693$

c) $\sin 64° = .8988$

$\cos 89° = .0175$

$\tan 13° = .2309$

d) $5 \sin 42° = 3.3455$

$3.5 \cos 13° = 3.4104$

$9 \tan 39° = 7.2882$

4. a) $\theta = 18°$

b) $\theta = 61°$

c) $\theta = 64°$

d) $\theta = 34°$

5. a) $m \angle A = 62°$

b) $m \angle Q = 74°$

c) $m \angle F = 37°$

d) $m \angle A = 23°$

6. a) D

b) B

c) 1) $\cos 0° = 1$

2) $\tan 0° = 0$

3) $\cos 60° = .5$

d) 1) $\tan 45° = 1$

2) $\sin 90° = 1$

3) $\sin 30° = .5$

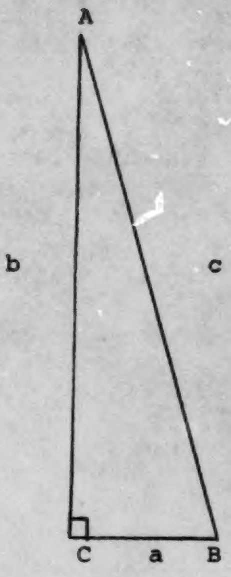
Answers (continued)

7. a) $\cos A \tan A = \frac{b}{c} \circ \frac{a}{b}$

$$= \frac{ab}{bc}$$

$$= \frac{a}{c}$$

$$= \sin A$$



b) $\frac{\sin A}{\cos A} = \frac{\frac{a}{c}}{\frac{b}{c}}$

$$= \frac{a}{c} \div \frac{b}{c}$$

$$= \frac{a}{c} \bullet \frac{c}{b}$$

$$= \frac{ac}{bc}$$

$$= \frac{a}{b}$$

$$= \tan A$$

c) $\frac{\sin A}{\tan A} = \frac{\frac{a}{c}}{\frac{a}{b}}$

$$= \frac{a}{c} \div \frac{a}{b}$$

$$= \frac{a}{c} \circ \frac{b}{a}$$

$$= \frac{ab}{ac}$$

$$= \frac{b}{c}$$

$$= \cos A$$

Answers (continued)

d) $\sin^2 A + \cos^2 A = (\frac{a}{c})^2 + (\frac{b}{c})^2$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$$

$$= \frac{a^2 + b^2}{c}$$

$$= \frac{c^2}{c^2}$$

$$= 1$$

8. a) $\sin 33° = \frac{AC}{24}$

$24 \cdot \sin 33 = AC$

$24 \cdot .5446 = AC$

$13.0704' = AC$

b) $\tan 67° = \frac{PR}{19}$

$19 \cdot \tan 67° = PR$

$19 \cdot 2.3559 = PR$

$44.7621 \text{ cm} = PR$

c) $\cos 70° = \frac{YZ}{23}$

$23 \cdot \cos 70° = YZ$

$23 \cdot .3420 = YZ$

$7.866\text{m} = YZ$

d) $\tan 27° = \frac{10.19}{AC}$ or

$AC \cdot \tan 27° = 10.19$

$AC = \frac{10.19}{\tan 27°}$

$AC = \frac{10.19}{.5095}$

$AC = 20"$

$\tan 63° = \frac{AC}{10.19}$

$10.19 \cdot \tan 63° = AC$

$10.19 \cdot 1.9626 = AC$

$19.998894 = AC$

Answers (continued)

9. a) $\sin 70° = \frac{60}{x}$

$x \sin 70° = 60$

$x = \frac{60}{\sin 70°}$

$x = \frac{60}{.9397}$

$x = 63.9'$

$\tan 70° = \frac{60}{x}$

$x \tan 70° = 60$

$x = \frac{60}{\tan 70°}$

$x = \frac{60}{2.7475}$

$x = 21.8$

b) $\sin \theta = \frac{200}{300}$

$\sin \theta = .6667$

$\theta = 42°$

c) $\sin 25° = \frac{50}{x}$

$x \sin 25° = 50$

$x = \frac{50}{\sin 25°}$

$x = \frac{50}{.4226}$

$x = 118.3'$

d) $\sin 42° = \frac{x}{3000}$

$3000 \cdot \sin 42° = x$

$3000 \cdot .6691 = x$

$2007.3' = x$

ENRICHMENT

UNIT XIII - PERIMETER, AREA, AND VOLUME

PURPOSE

Students completing algebra in the 8th grade often miss additional information about perimeter, area, and volume. Exposure to such material is very important for the students to have in order to progress in geometry. This unit provides much of this background information.

OVERVIEW

The emphasis in this unit is placed on the students' being introduced to area and volume formulas and to calculating areas and volumes by substituting into these formulas. The derivation and proof of these formulas is left for study in a course on geometry.

SUGGESTIONS TO THE TEACHER

Instructional Days: 8-10
Minimal Course Objectives: Numbers 1-3
Average Course Objectives: Numbers 1-6
Maximal Course Objectives: ALL

The formulas may be used to review solving literal equations:

Example: Solve $SA = 2\pi r^2 + 2\pi rh$ for h

VOCABULARY

altitude
area
base
circumference
cone
cube
cylinder
depth
diameter
face
height
lateral area
length
parallelogram
perimeter
pi
polygon
prism
pyramid
radius
rectangle
regular
rhombus
slant height
sphere
square
surface area
traezoid
triangle
vertex
volume
width

ENRICHMENT

UNIT XIII - PERIMETER, AREA, AND VOLUME

PERFORMANCE OBJECTIVES

1. Compute the perimeter of a given polygon. (II)
2. Compute the circumference of a circle. (II)
3. Compute the area of each of the following: triangle, square, rectangle, parallelogram, trapezoid, and circle. (II)
4. Compute the lateral area of each of the following: rectangular prism, cylinder, cone. (II)
5. Compute the surface area of the following: rectangular prism, cylinder, sphere, cone. (II)
6. Compute the volume of each of the following: rectangular prism, triangular prism, cylinder, sphere, rectangular pyramid, cone. (II)
7. Compute the area of a geometrical figure composed of triangles, squares, rectangles, parallelograms, trapezoids, and/or circles. (III)
8. Compute the volume of a geometrical solid composed of rectangular prisms, rectangular pyramids, cones, cylinders, spheres, and/or triangular prisms. (III)

Minimal	Average	Maximal
#1 - 3	#1 - 6	All

UNIT XIII - PERIMETER, AREA, AND VOLUME

CROSS REFERENCES

TEXTS (BY AUTHOR)

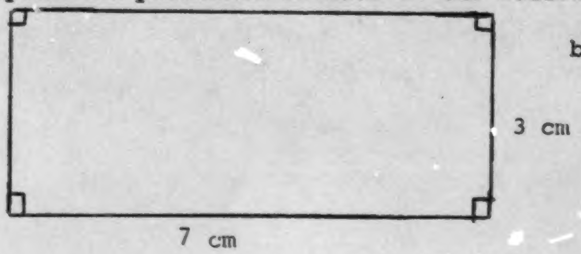
Objective	Dolciani et al (1976)	Payne et al (1977)	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Jacobs (Part II) (1976)	Johnson et al (Part I) (1977)
1	12-13	2	---	14-16 180-181	67	---
2	12-13	9	402-403	14-16	---	---
3	13-14	2,6,9	---	14-16	71	222,217, 13,218,307
4	---	---	---	---	---	---
5	---	---	---	---	---	---
6	14	8	---	---	---	14,222 222
7	15	----	---	---	---	---
8	---	---	---	---	---	---

PERFORMANCE OBJECTIVE XIII-1

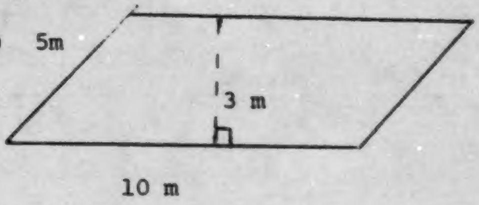
Compute the perimeter of a given polygon.

Compute the perimeter of each of the following:

a)



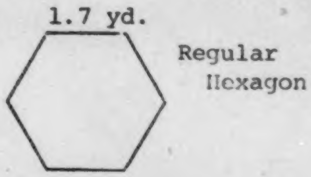
b)



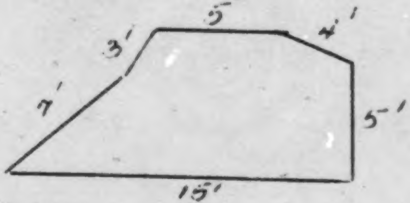
Answer __________

Answer __________

c)


1.7 yd.
Regular
Hexagon

d)



Answer __________

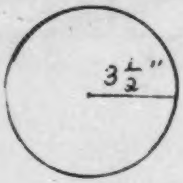
Answer __________

PERFORMANCE OBJECTIVE XIII-2

Compute the circumference of a circle.

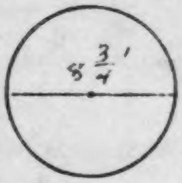
Compute the circumference for each of the following:

a) Use $\Pi = \frac{22}{7}$



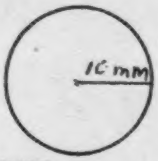
Answer __________

b) Use $\Pi = \frac{22}{7}$



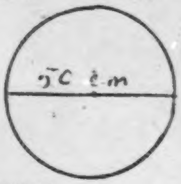
Answer __________

c) Use $\Pi = 3.14$



Answer __________

d) Use $\Pi = 3.14$



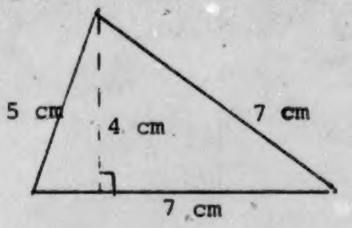
Answer __________

PERFORMANCE OBJECTIVE XIII-3

Compute the area of each of the following: triangle, square, rectangle, parallelogram, trapezoid, circle.

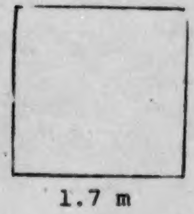
Compute the area of each of the following:

a)


5 cm
4 cm
7 cm
7 cm

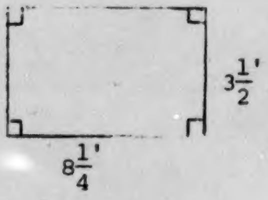
Answer ____________

b) Square



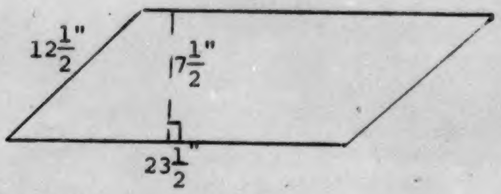
Answer ____________

c)



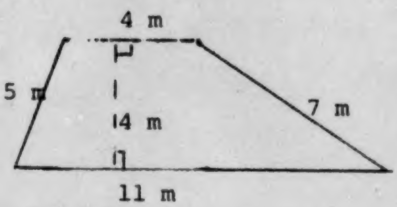
Answer ____________

d) Parallelogram



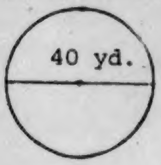
Answer ____________

e)


4 m
5 m
4 m
7 m
11 m

Answer

f) Use Π = 3.14



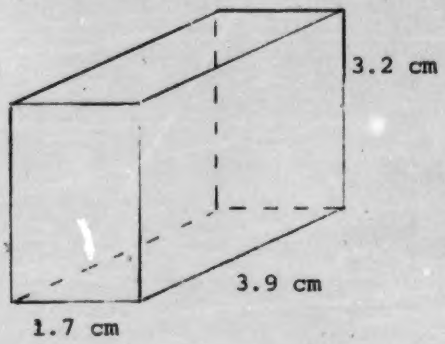
Answer ____________

PERFORMANCE OBJECTIVE XIII-4

Compute the lateral area of each of the following: rectangular prism, cylinder, cone.

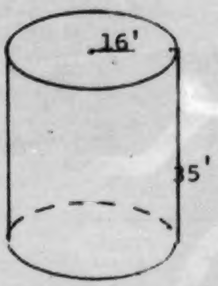
Compute the lateral area for each of the following:

a)


3.2 cm
3.9 cm
1.7 cm

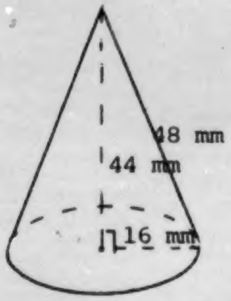
Answer ________

b) Use $\pi = 3.14$



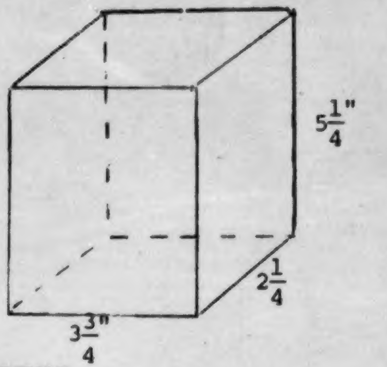
Answer ________

c) Use $\pi = 3.14$


48 mm
44 mm
16 mm

Answer ________

d)


$5\frac{1}{4}$"
$2\frac{1}{4}$
$3\frac{3}{4}$"

Answer ________

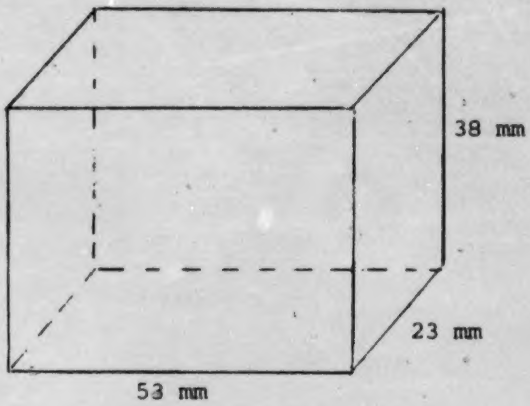
316

PERFORMANCE OBJECTIVE XIII-5

Compute the surface area of the following: rectangular prism, cylinder, sphere, cone.

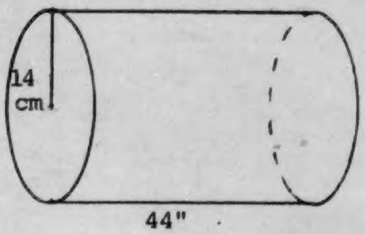
Compute the surface area for each of the following:

a)


38 mm
23 mm
53 mm

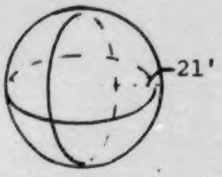
Answer ________

b) Use $\Pi = \frac{22}{7}$



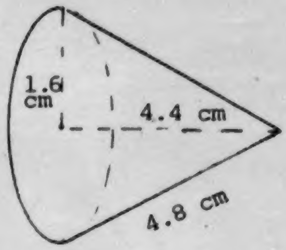
Answer ________

c) Use $\Pi = \frac{22}{7}$



Answer ________

d) Use $\Pi = 3.14$

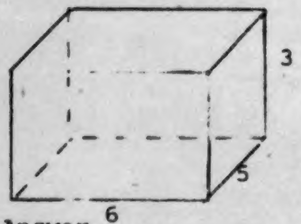

1.6 cm
4.4 cm
4.8 cm

Answer ________

PERFORMANCE OBJECTIVE XIII-6

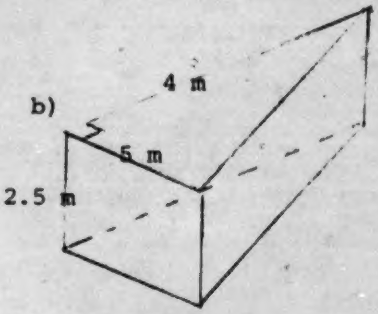
Compute the volume of each of the following: rectangular prism, cylinder, sphere, rectangular pyramid, cone.

a)



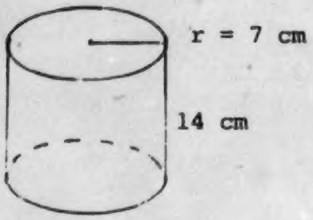
Answer __________

b)



Answer __________

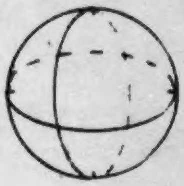
c)



$\pi = 3.14$

Answer __________

d)

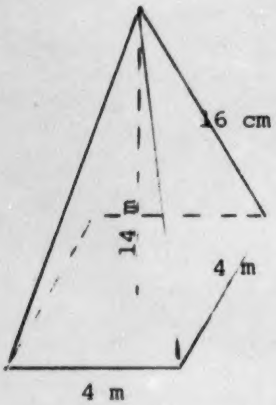


r = 35'

$\pi = \frac{22}{7}$

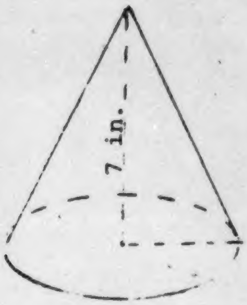
Answer __________

e)


16 cm
14 m
4 m
4 m

Answer __________

f)



r = 3 in.

$\pi = 3.14$

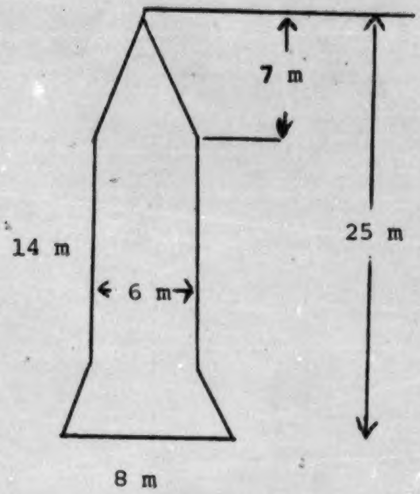
Answer __________

PERFORMANCE OBJECTIVE XIII-7

Compute the area of a geometrical figure composed of triangles, squares, rectangles, parallelograms, trapezoids and/or circles.

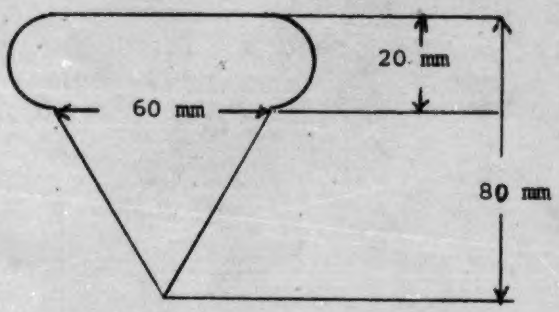
Compute the area for each of the following:

a)


7 m
14 m
25 m
6 m
8 m

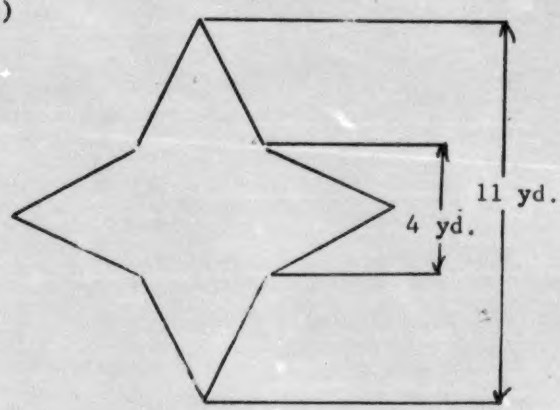
Answer ________

b) Use $\pi = 3.14$


20 mm
60 mm
80 mm

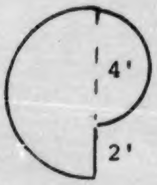
Answer ________

c)



Answer ________

d) Use $\pi = 3.14$



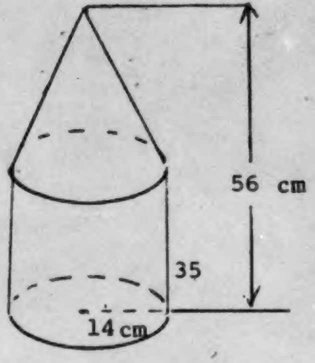
Answer ________

PERFORMANCE OBJECTIVE XIII-8

> Compute the volume of a geometrical solid composed of rectangular prisms, rectangular pyramids, cones, cylinders, spheres, and/or triangular prisms.

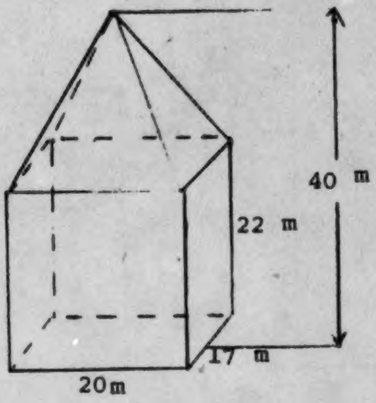
Compute the volume of the following:

a) Use $\Pi = \frac{22}{7}$


56 cm
35
14 cm

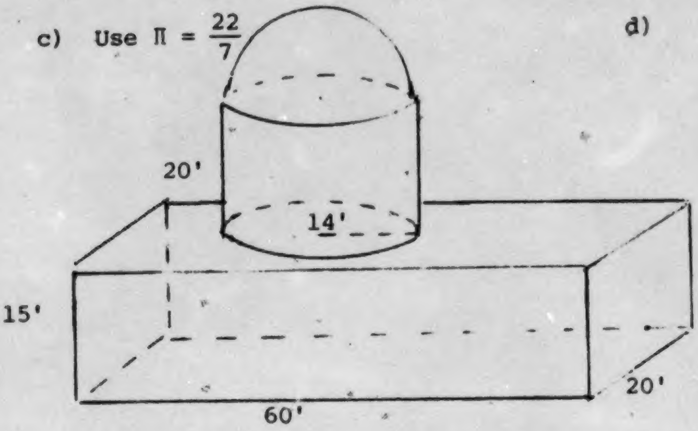
Answer ____________

b)


40 m
22 m
17 m
20m

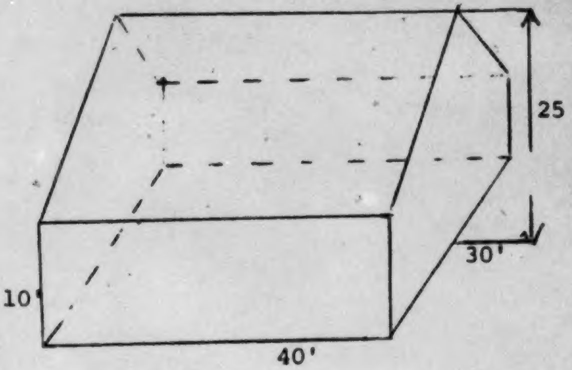
Answer ____________

c) Use $\Pi = \frac{22}{7}$


20'
14'
15'
60'
20'

Answer

d)



Answer __________

UNIT XIII - PERIMETER, AREA, AND VOLUME

Answers

1. a) 20 cm

b) 30 m

c) 10.2 yd.

d) 39'

2. a) 22"

b) $27\frac{1}{2}$'

c) 62.8 mm

d) 157 cm

3. a) 14 cm^2

b) 2.89 m^2

c) $28\frac{7}{8}$ sq. ft.

d) $176\frac{1}{4}$ sq. in.

e) 30 m^2

f) 1256 sq. yd.

4. a) 35.84 cm^2

b) 3516.8 sq. ft

c) 2411.52 mm^2

d) 63 sq. in.

5. a) 8214 mm^2

b) 5104 sq. in.

c) 5544 sq. ft.

d) 32.1536 cm^2

6. a) 90 cu. in.

b) 25 m^3

c) 2154.04 cm^3

d) $179{,}666.6\overline{6}$ cu. ft.

e) $74.\overline{6}$ m^3

f) 65.94 cu. in.

7. a) 133 m^2

b) 3314 mm^2

c) 44 sq. yd.

d) 20.41 sq. ft.

8. a) 25,872 cm^3

b) 9520 m^3

c) $36{,}069\frac{1}{3}$ cu. ft.

d) 21,000 cu. ft.

ENRICHMENT

UNIT XIV - RATIO, PROPORTION, AND PERCENT

PURPOSE

This unit is intended to provide a review of percent and to give students practice in solving practical consumer related problems. The information is especially important to 8th grade algebra students who would ordinarily receive additional instruction in these topics in the regular 8th grade curriculum.

OVERVIEW

Proportions and their relation to percent are the key to this unit. Discount, commission, and tax problems may be discussed by adapting the proportion $\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$ to these kinds of problems or by the traditional method of convering the percent to a decimal and multiplying. The interest formula and its applications to simple and compound interest are also discussed.

SUGGESTIONS TO THE TEACHER

Instructional Days: 7-9
Minimal Course Objectives: Numbers 1-8
Average Course Objectives: ALL
Maximal Course Objectives: ALL

Administration of a pretest before beginning this unit could be important. It may be desired to integrate this unit with Unit IX on Rational Algebraic Expressions.

VOCABULARY

base
commission
compound interest
decimal numeral
discount
extremes
interest
means

percent
percentage
principal
proportion
ratio
simple interest
term

ENRICHMENT

UNIT XIV - RATIO, PROPORTION, AND PERCENT

PERFORMANCE OBJECTIVES

1. Write a proportion which represents the relationship between quantities in a given word problem. (III)
2. Solve word problems using proportions. (III)
3. Write a given percent as a ratio. (II)
4. Write a given percent as a decimal numeral. (II)
5. Find the percentage of a given number using the proportion: $\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$. (III)
6. Find the base, using the proportion: $\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$. (III)
7. Find the percent, using the proportion: $\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$. (III)
8. Compute simple interest. (II)
9. Compute compound interest. (II)
10. Solve word problems that involve discounts. (III)
11. Solve word problems that involve commissions. (III)
12. Solve word problems that involve taxes. (III)
13. Solve word problems that involve percent mixtures. (III)

Minimal	Average	Maximal
#1-8	All	All

UNIT XIV - RATIO, PROPORTION, AND PERCENT

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Keedy et al (1978)	Payne et al (1977)	Sobel Banks (1977)	Travers et al (1977)
1	228-229	225-228	276-278	158-161	363-368	158-160	364-366	481-484
2	231	225-228	278	162	363-369	158-162	367-368	481-484
3	228	375	---	163-164	---	---	---	293-294
4	228	375	---	163-164	---	---	---	---
5	229	375-377	276-277	165-166	---	---	---	---
6	229	375-377	---	165-166	---	---	---	---
7	229	375-377	---	165-166	---	---	---	---
8	236-239	377-378	282-283	---	---	---	---	103-105
9	---	---	---	---	---	---	---	---
10	---	377-378	---	167-168 170-171	---	---	---	---
11	---	377-378	---	---	---	---	---	---
12	---	377-378	---	---	---	---	---	---
13	234-235	378	279-282	---	266-268	---	---	---

UNIT XIV - RATIO, PROPORTION, AND PERCENT

CROSS REFERENCES

TEXTS (BY AUTHOR)

Objective	Denholm et al Part I - '77	Denholm et al Part II -'77	Jacobs (Part I) (1976)	Johnson et al (Part I) (1977)
1	---	---	144-147	78-82
2	---	---	144-147	78-82
3	74-76	178-18	148-151	83-87
4	74-76	178-180	148-151	85,86,88
5	---	181-183	152-159	89-95
6	---	181-183	152-159	89-95
7	---	181-183	152-159	89-95
8	---	---	160-163	218-219
9	---	---	---	---
10	---	---	164-168	---
11	---	---	---	---
12	---	---	---	---
13	---	185-186	196-199	---

PERFORMANCE OBJECTIVE XIV-1

Write a proportion which represents the relationship between quantities in a given word problem.

a) Henry and Louise drove 510 miles one day of their vacation. They estimated this was done in $8\frac{1}{2}$ hours. At the same average rate, how long will it take them to drive an additional 720 miles?

Answer __________

b) When buying groceries, Rose bought 6 limes for 39¢. Ralph needed 10 limes to make punch. How much would they cost?

Answer __________

c) A crepe mix calls for $1\frac{3}{4}$ cups of milk for 2 cups of mix. How much milk will be needed for 3 cups of mix?

Answer __________

d) A five acre field yields 93 bushels of peanuts. About how many bushels would an 18 acre field yield under similar conditions?

Answer __________

PERFORMANCE OBJECTIVE XIV-2

Solve word problems using proportions.

a) The straight line distance between Richmond and Baltimore is 140 miles. How far apart should they be placed on a map on which $\frac{1''}{2}$ represents 45 miles?

Answer __________

b) Mary Ella, the pharmacist, had placed an order with the warehouse for 81 dsl tablets which cost $11.34. How much will it cost the pharmacy when she fills Wes' order for 13 tablets?

Answer __________

c) The differential gear in an automobile turns twice for every 7 turns of the rear wheels. If the differential gear turns 358 times, how many times do the rear wheels turn?

Answer __________

d) In Mr. Smith's math classes, 13 students out of 35 fail. If Mr. Smith has 145 students this year, how many will fail (to the nearest student)?

Answer __________

PERFORMANCE OBJECTIVE XIV-3

Write a given percent as a ratio.

a) 33% is a little more than _________.

A. $\frac{3}{8}$

B. $\frac{2}{5}$

C. $\frac{3}{10}$

D. None of the above

Answer _________

b) Write a fraction for each percent.

A. 50% = _________

B. $66\frac{2}{3}$ % = ________

C. $16\frac{2}{3}$ % = ________

c) Which of the following percents is represented by the fraction $\frac{1}{8}$?

A. $12\frac{1}{2}$ %

B. $\frac{1}{8}$ %

C. 80%

D. None of the above

Answer _________

d) 150% equals _________.

A. $\frac{3}{20}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. None of the above

Answer _________

PERFORMANCE OBJECTIVE XIV-4

Write a given percent as a decimal numeral.

a) 55% is a little less than:

A. .549

B. .6

C. .5

D. All of the above

Answer __________

b) Write a decimal for each percent.

A. $66\frac{2}{3}$ % = __________

B. 25% = __________

C. $83\frac{1}{3}$ % = __________

c) $\frac{1}{2}$ % equals:

A. $.0\frac{1}{2}$

B. .5

C. .005

D. None of the above

Answer __________

d) Which one of the following equals 250%?

A. 2.5

B. .25

C. .025

D. All of the above

Answer __________

PERFORMANCE OBJECTIVE XIV-5

Find the percentage of a given number, using the proportion:

$\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$.

a) 52% of 72 is what number?

Answer __________

b) What is 45% of 135?

Answer __________

c) 70% of 800 equals __________.

A. 56

B. 560

C. 5600

D. None of the above

Answer __________

d) 175% of 80 is __________.

A. 1.4

B. 14

C. 140

D. None of the above

Answer __________

PERFORMANCE OBJECTIVE XIV-6

Find the base using the proportion: $\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$

a) 38% of what number is 95 ?

Answer __________

b) 96 is 24% of what number?

Answer __________

c) 76 is 80% of what number?

A. 9.5

B. 95

C. 950

D. None of the above

Answer __________

d) 120% of what number is 5.4?

A. 4.5

B. 450

C. 4500

D. None of the above

Answer __________

PERFORMANCE OBJECTIVE XIV-7

Find the percent, using the proportion: $\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$.

a) 37 is what percent of 222?

Answer __________

b) What percent of 112 is 70?

Answer __________

c) 7.2 is what percent of 80?

A. 9%

B. 90%

C. 900%

D. None of the above

Answer __________

d) 160 is what percent of 16?

A. 1%

B. 10%

C. 100%

D. 1000%

Answer __________

PERFORMANCE OBJECTIVE XIV-8

Compute simple interest.

a) John deposits $700 in a bank which pays simple interest at a rate of 5% per year. How much interest will John receive after 9 months?

Answer __________

b) What is the simple interest on $1700 after 1 year 8 months at a rate of 6% per year?

Answer __________

c) In order to make a down payment on an apartment building, Pearl borrowed $10,000 from the Credit Union. It charges interest at a rate of 12% per year. How much simple interest will she be charged if she pays the money back after 1 month?

Answer __________

d) Don Kowski borrows $500 from his good friend Kay for 8 months and agrees to pay her simple interest at an 8% rate. How much interest will Don owe?

Answer __________

PERFORMANCE OBJECTIVE XIV-9

Compute compound interest.

a) Chuck deposited $300 in a savings account at the Price National Bank. If he receives interest at a rate of 5% per year compounded quarterly, how much will be in Chuck's account after 9 months?

Answer __________

b) Compute the total interest paid on a savings account of $10,000 after 1 year if interest is paid at a rate of 6% per year compounded quarterly.

Answer __________

c) When David was 16, his grandfather gave him $3000. He deposited it in the MR National Bank. If the interest rate was 5% per year compounded semiannually, how much was in his account on his 18th birthday?

Answer __________

d) Find the total interest paid on a $500 savings account after 3 years at 5% per year compounded semiannually.

Answer __________

PERFORMANCE OBJECTIVE XIV-10

Solve word problems that involve discounts.

a) Mary bought a mink coat on sale for 45% off. If the coat usually cost $3500, what did Mary pay for the coat?

Answer __________

b) At their going-out-of-business sale, Schmitt's Sporting Goods Store offered discounts of 30-70%. If Ruth bought a pair of skis at 65% off and they normally cost $150, how much did she pay for the skis?

Answer __________

c) In August one year all bathing suits at the Angel's Rest store were sold at 75% of their marked price. What was the rate of discount? What was the amount of discount on a Madame Elaine bathing suit marked $32?

Answer __________

d) At Weikel's TV shop, a Webber TV regularly priced $495 is on sale for 20% off. The same model at Toby's TV Tent has an original price of $455 but now is discounted 10%. Which television would be the better buy?

Answer __________

PERFORMANCE OBJECTIVE XIV-11

Solve word problems that involve commissions.

a) Wes sold shoes to pay his way through school. He was paid a salary plus a bonus commission of 5% of his sales over $200 each week. How much was his bonus in a week during which he sold $570 worth of shoes?

Answer __________

b) Ms. Rosas received a commission of 20% for obtaining a rare antique for Mr. Webb. If the antique cost $525, how much did Ms. Rosas receive for her commission?

Answer __________

c) One summer, Helen drove a Bad Joke Ice Cream truck. She was to receive a 2% commission on all sales over $350 each week. The last week of the summer, she had a total sales of $1126.32. What was her commission for the week?

Answer __________

d) Mr. Balet is an agent for the superstar basketball player Mitch Donkowski. For negotiating a $1,200,000 six year no-cut contract with the Bullets, Mr. Balet was given a 15% commission. How much did he receive?

Answer __________

PERFORMANCE OBJECTIVE XIV-12

Solve word problems that involve taxes.

a) At her summer job, Susan earned $183.05 in a week. From this, various amounts of money were withheld to cover her federal income tax, state income tax, and social security. 6.13% of her total weekly earnings were withheld to cover her social security payment. How much money was withheld for social security? (Round to nearest cent.)

Answer __________

b) In 1973 Martha earned $17,500. When she computed her federal income tax, she read in the tax table that people who earned over $16,000 but not over $18,000 had to pay $4,330 plus 42% of the amount they made over $16,000. How much federal income tax did Martha have to pay that year?

Answer __________

c) The state tax table in Maryland for the year 1978 requires that the amount of tax to be paid is $90 plus 5% of all taxable income over $5000. If Ralph's taxable income was $11,854, how much state tax did he pay?

Answer __________

d) Henry bought a Sunpanic television for $495 and a stand for $39.95. He lives in Maryland which has a 5% sales tax. What was the total amount paid?

Answer __________

PERFORMANCE OBJECTIVE XIV-13

Solve word problems that involve percent mixtures.

a) A chemist has a solution that is 35% acid. She wishes to increase the amount of acid to 75% by adding 90% acid. If she starts with 900 ml of the 35% solution, how much of the 90% acid must be added to have the desired result?

Answer __________

b) Winter birdseed contains 60% sunflower seeds and summer birdseed contains only 25% sunflower seeds. How much of each would you use to make 28 kg of birdseed containing 50% sunflower seeds?

Answer __________

c) How many grams of a 65% solution of sodium hydroxide must be mixed with 160 grams of a 30% solution of sodium hydroxide to make a 55% solution of sodium hydroxide?

Answer __________

d) In creating his monster, Dr. Frankenstein kept the brain in a 6% salt solution. He sent his incompetent assistant Igor to the pharmacist to buy 500 ml of this solution. Igor bought 200 ml of 4% solution and 300 ml of 7% solution by mistake. How much of the 4% solution must he mix with the 300 ml of 7% solution to form a 6% solution?

Answer __________

UNIT XIV - RATIO, PROPORTION, AND PERCENT

Answers

1. a) $\frac{510}{8\frac{1}{2}} = \frac{720}{x}$

   b) $\frac{6}{39} = \frac{10}{x}$

   c) $\frac{1\frac{3}{4}}{2} = \frac{x}{3}$

   d) $\frac{5}{93} = \frac{18}{x}$

2. a) $\frac{140}{x} = \frac{45}{\frac{1''}{2}}$

   $x = 1\frac{5''}{9}$

   b) $\frac{81}{\$11.34} = \frac{13}{x}$

   $x = \$1.82$

   c) $\frac{2}{7} = \frac{358}{x}$

   $x = 1253$

   d) $\frac{13}{35} = \frac{x}{145}$

   $x = 54$

3. a) C

   b) A. $\frac{1}{2}$

   B. $\frac{2}{3}$

   C. $\frac{1}{6}$

   c) A

   d) C

4. a) B

   b) A. $.66\overline{6}$

   B. $.25$

   C. $.83\overline{3}$

   c) C

   d) A

5. a) 37.44

   b) 60.75

   c) B

   d) C

6. a) 250

   b) 400

   c) B

   d) A

7. a) $16\frac{2}{3}\%$

   b) $62\frac{1}{2}\%$

   c) A

   d) D

8. a) $26.25

   b) $170

   c) $100

   d) $26.67

Answers (continued)

9. a) $311.39
   b) $613.64
   c) $3311.44
   d) $79.85

10. a) $1925
    b) $52.50
    c) $24
    d) Weikel's TV ($495 reg/$396 sale) is better buy.

11. a) $18.50
    b) $105
    c) $15.53
    d) $180,000

12. a) $11.22
    b) 4960
    c) $432.70
    d) $561.70

13. a) 2400 ml of 90% acid
    b) 20 kg of 60% seed and 8 kg of 25% seed
    c) 400 g of 65% sodium hydroxide
    d) 150 ml of 4% solution

ENRICHMENT

UNIT XV - FLOWCHARTS

PURPOSE

Computers are becoming more and more a part of our everyday life. Many of today's algebra students will be working with computers tomorrow. To begin preparing them to understand computers, this unit has been included. The ability to draw a flowchart for the step-by-step solution of a problem is a valuable aide to programming.

OVERVIEW

The four basic flowcharting symbols are introduced: Start/Stop, Input/Output, Operation, and Decision. (The symbols used in this unit are those used in Coan's Basic BASIC and Golden's Computer Programming in the BASIC Language. These books are commonly used in teaching programming in the county.) Instruction follows in order of increasing complexity from inserting the steps of a given algorithm into its flowchart to writing an algorithm and drawing its flowchart.

SUGGESTIONS TO THE TEACHER

Instructional Days: 7-9

Minimal Course Objectives: None

Average Course Objectives: 1-3

Maximal Course Objectives: ALL

VOCABULARY

algorithm
decision box
flowchart
input/output box
loop
operation

ENRICHMENT

UNIT XV - FLOWCHARTS

PERFORMANCE OBJECTIVES

1. Identify the four basic flowchart symbols: Start/Stop, Input/Output, Operation, and Decision. (I)
2. Complete a blank flowchart when given the necessary instructions out of sequence. (II)
3. Determine the output for a given input by following a given flowchart. (III)
4. Draw the flowchart for a given sequence of instructions. (III)
5. Construct a flowchart which specifies the procedure for accomplishing a given task. (III)

Minimal	Average	Maximal
None	#1 - 3	All

UNIT XV - FLOWCHARTS

CROSS REFERENCES

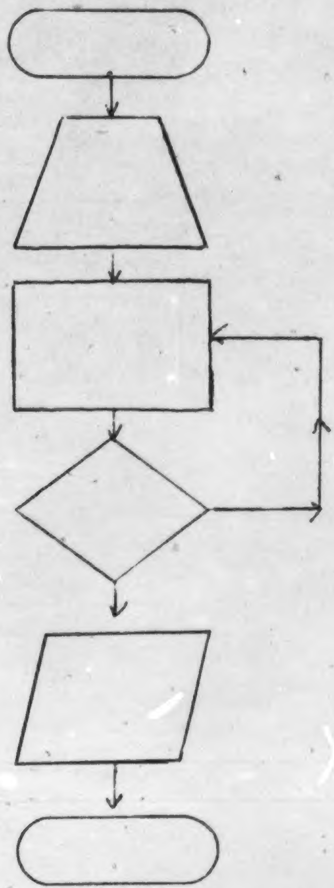
TEXTS (BY AUTHOR)

Objective	Dolciani et al (1976)	Dolciani et al (1978)	Foster et al (1979)	Jacobs (1974)	Sobel Banks (1977)	Travers et al (1977)
1	---	1-2	21	543	100-101	---
2	---	---	---	---	---	---
3	388	3-6	32	544-546	---	---
4	---	---	---	---	---	---
5	---	---	---	547	101	148

PERFORMANCE OBJECTIVE XV-1

Identify the four basic flowchart symbols (Start/Stop, Input/Output, Operation, and Decision).

a) In each box, write the name of the symbol.



1.

2.

3.

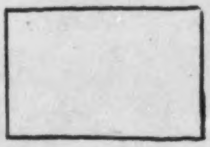
4.

5.

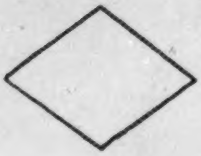
6.

b) Match each symbol with its name.

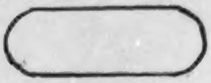
____ 1.



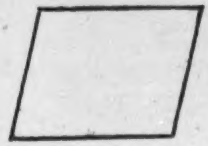
____ 2.



____ 3.



____ 4.



A. Decision

B. Input/Output

C. Operation

D. Start/Stop

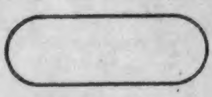
PERFORMANCE OBJECTIVE XV-1 (continued)

c) Draw the symbol for each one of the following:

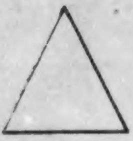
1. Start/Stop
2. Operation
3. Input/Output
4. Decision

d) Which of the following is not a flowchart symbol?

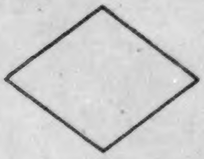
A.



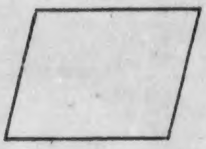
B.



C.



D.



Answer ________________

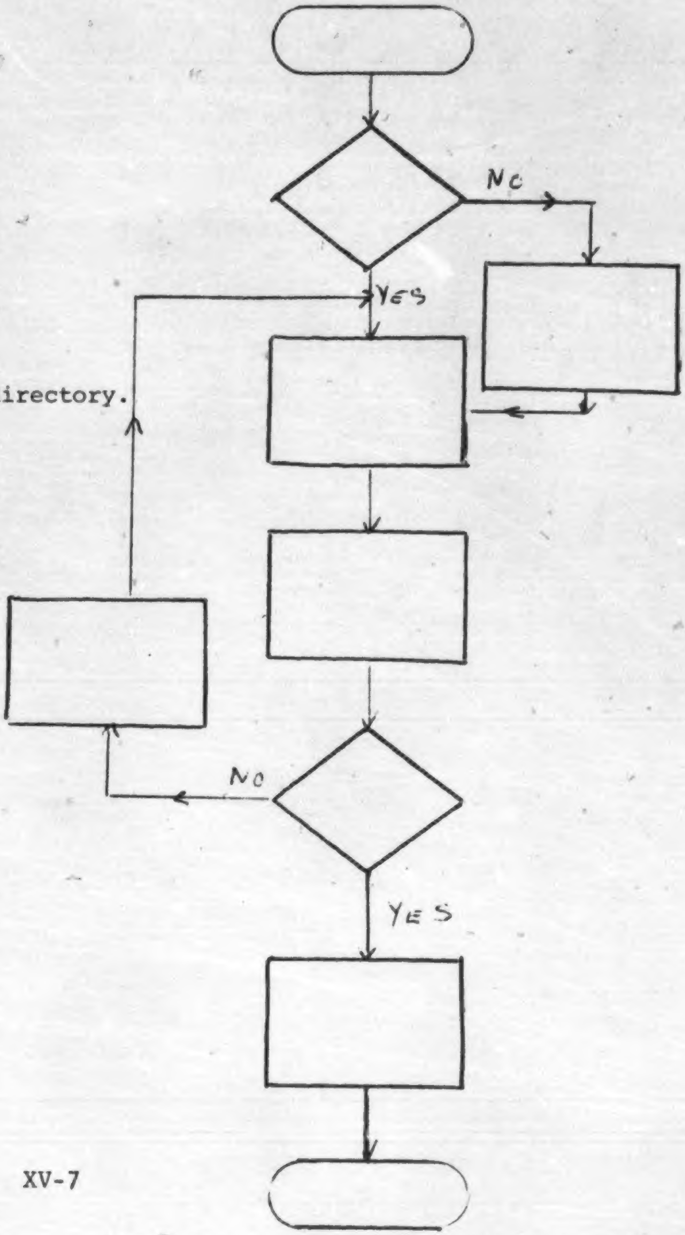
PERFORMANCE OBJECTIVE XV-2

Complete a blank flowchart when given the necessary instructions out of sequence.

a) In the appropriate box, place the number corresponding to each instruction.

Making a Phone Call

1. Pick up receiver.
2. Say "Hello!"
3. Does someone answer?
4. Dial the number.
5. Hang up.
6. Do you know the number?
7. Find the number in the directory.
8. Start.
9. Stop.



PERFORMANCE OBJECTIVE XV-2 (continued)

b) In the appropriate box, place the number corresponding to each instruction.

Selecting a TV Program

1. Do you want to see this show?

2. Do you know what show you want to see?

3. Start.

4. Stop.

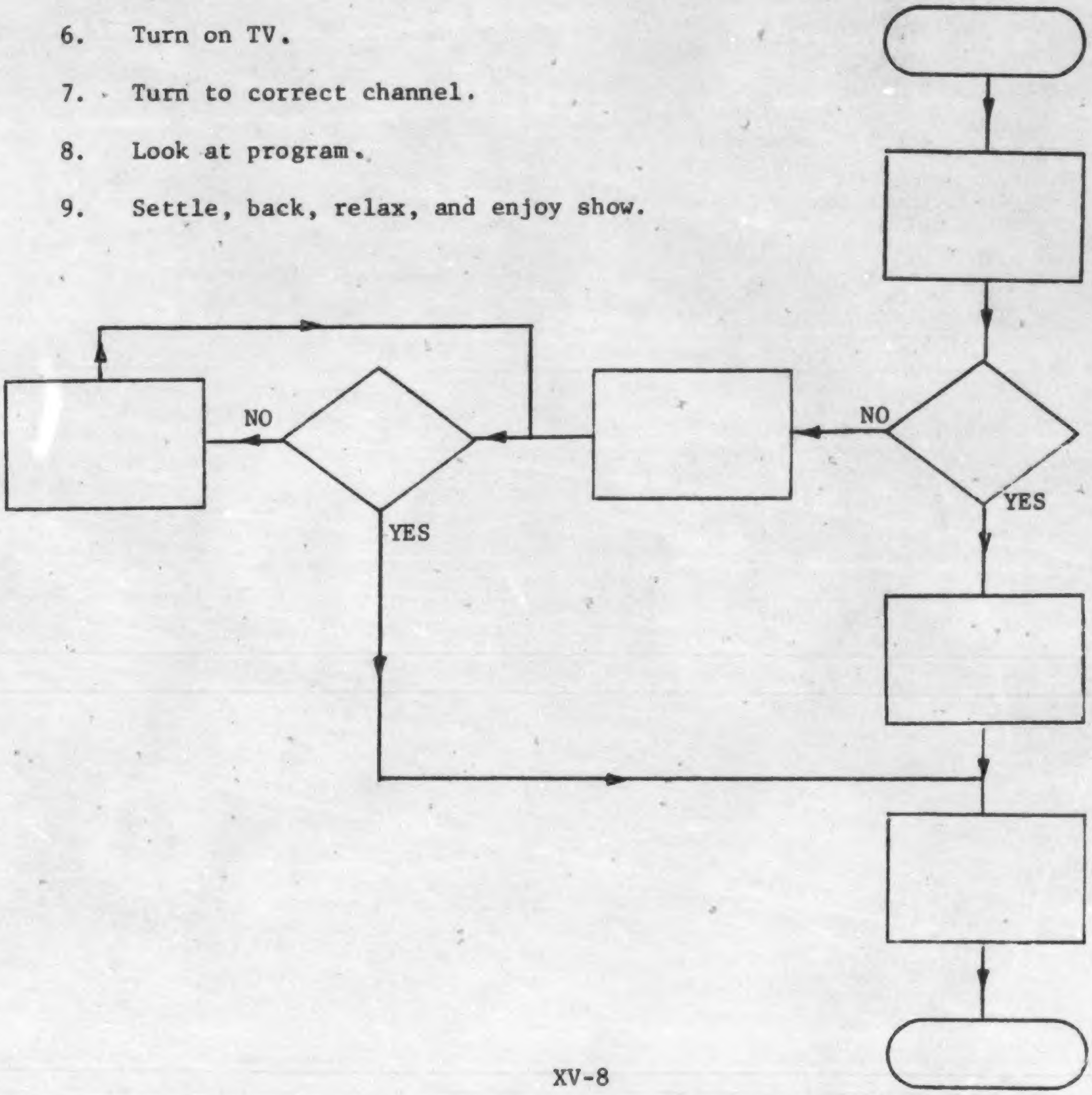
5. Change to next channel.

6. Turn on TV.

7. Turn to correct channel.

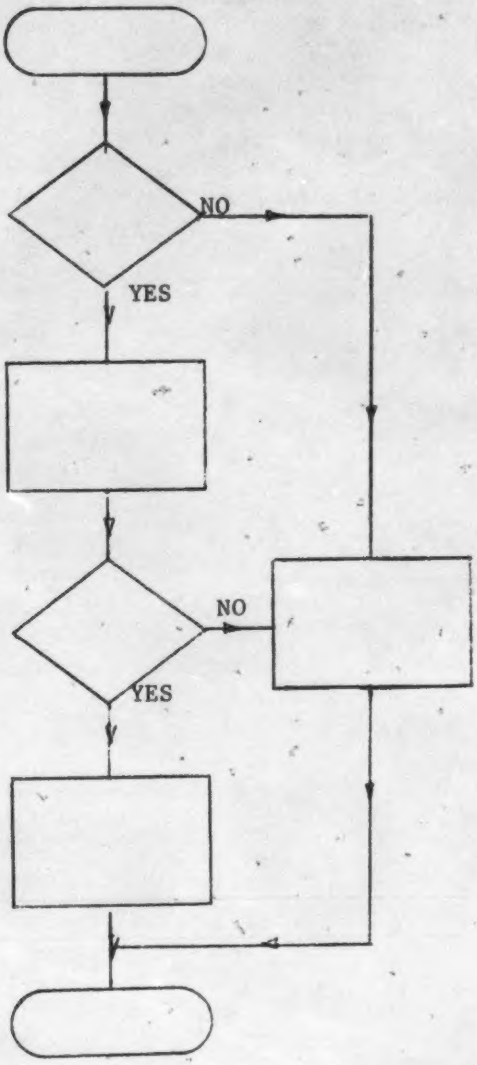
8. Look at program.

9. Settle, back, relax, and enjoy show.



PERFORMANCE OBJECTIVE XV-2 (continued)

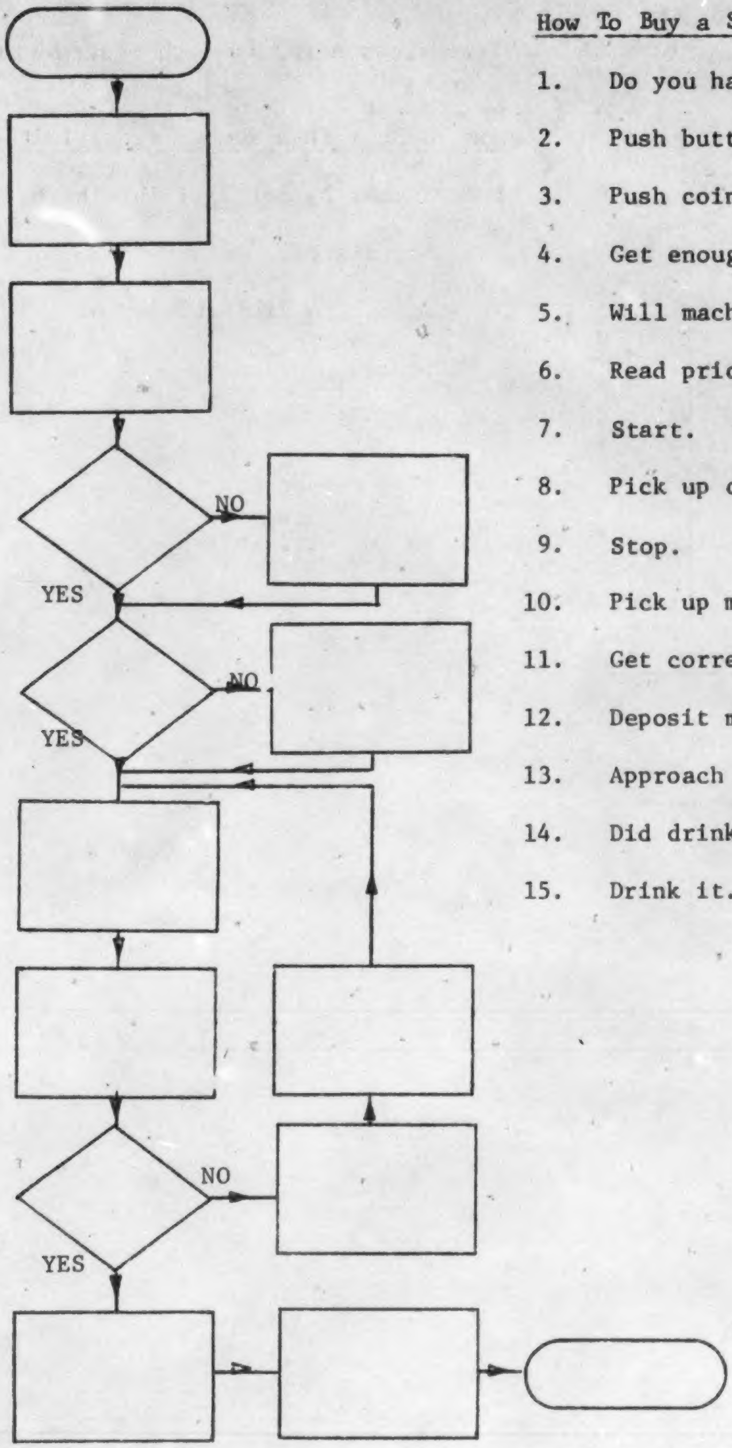
c) In the appropriate box, place the number corresponding to each instruction.



How To Tell if a Number is Divisible by 6.

1. Number is not divisible by 6.
2. Add digits.
3. Number is divisible by 6.
4. Start.
5. Stop.
6. Does number end in 0, 2, 4, 6, or 8?
7. Is sum divisible by 3?

d) In the appropriate box, place the number corresponding to each instruction.



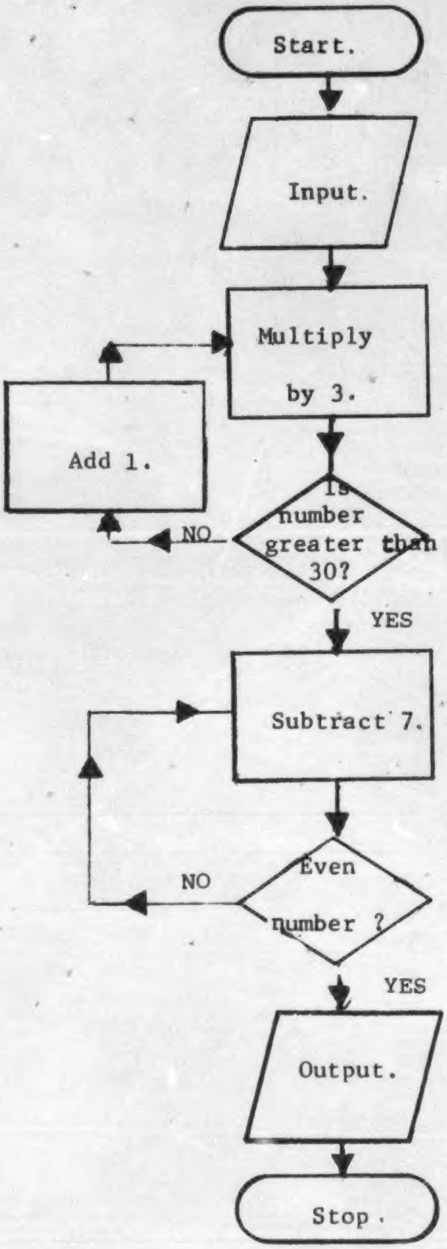
How To Buy a Softdrink from a Machine

1. Do you have enough money?
2. Push button.
3. Push coin return.
4. Get enough money.
5. Will machine give change?
6. Read price.
7. Start.
8. Pick up can.
9. Stop.
10. Pick up money from slot.
11. Get correct change.
12. Deposit money.
13. Approach machine.
14. Did drink come out?
15. Drink it.

PERFORMANCE OBJECTIVE XV-3

Determine the output for a given input by following a given flowchart.

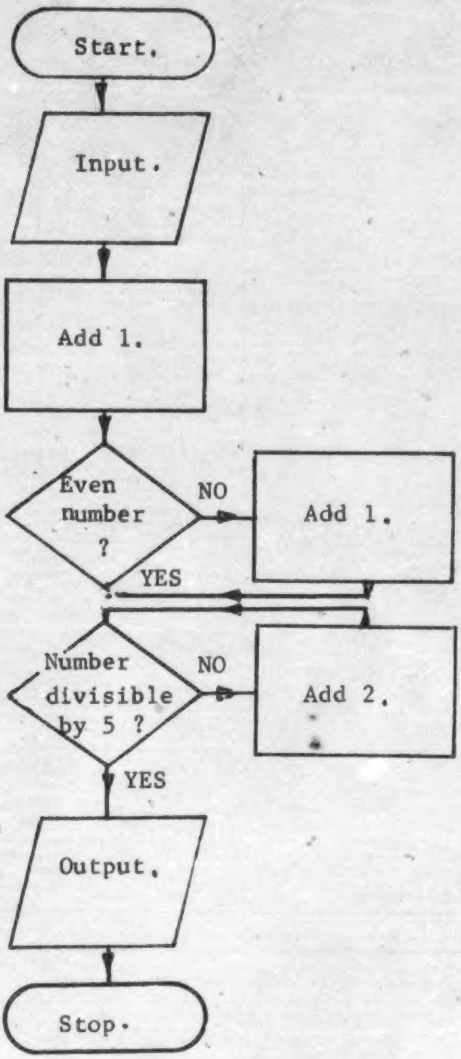
a) Given the following flowchart and Input, find the Output.


Start.
Input.
Multiply by 3.
Add 1.
NO
Is number greater than 30?
YES
Subtract 7.
NO
Even number ?
YES
Output.
Stop.

	Input	Output
1.	9	
2.	4	
3.	11	

PERFORMANCE OBJECTIVE XV-3 (continued)

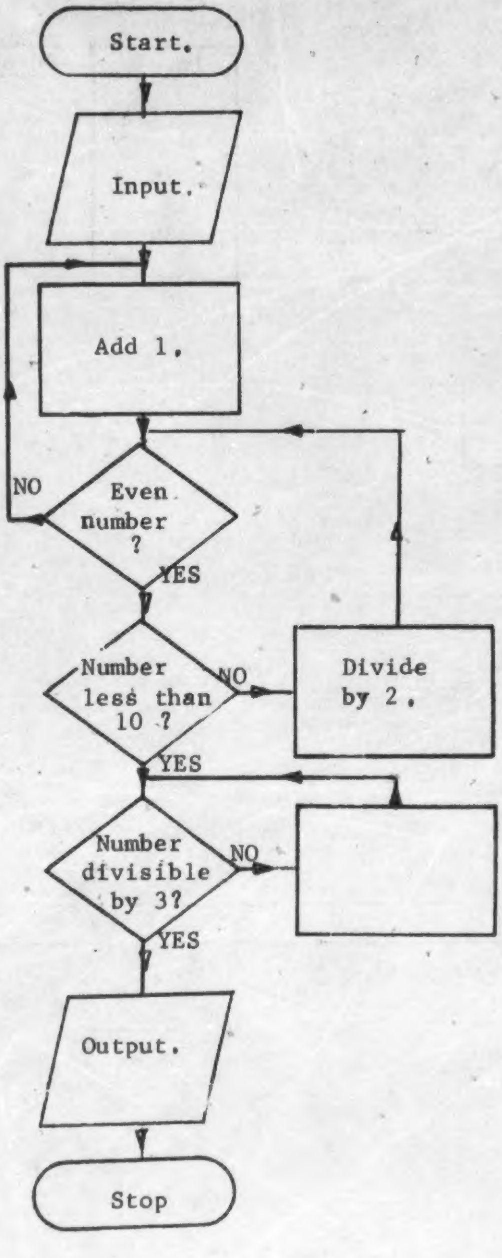
b) Given the following flowchart and Input, find the Output.


Start.
Input.
Add 1.
Even number ?
NO
Add 1.
YES
Number divisible by 5 ?
NO
Add 2.
YES
Output.
Stop.

	Input	Output
1.	1	
2.	13	
3.	24	

PERFORMANCE OBJECTIVE XV-3 (continued)

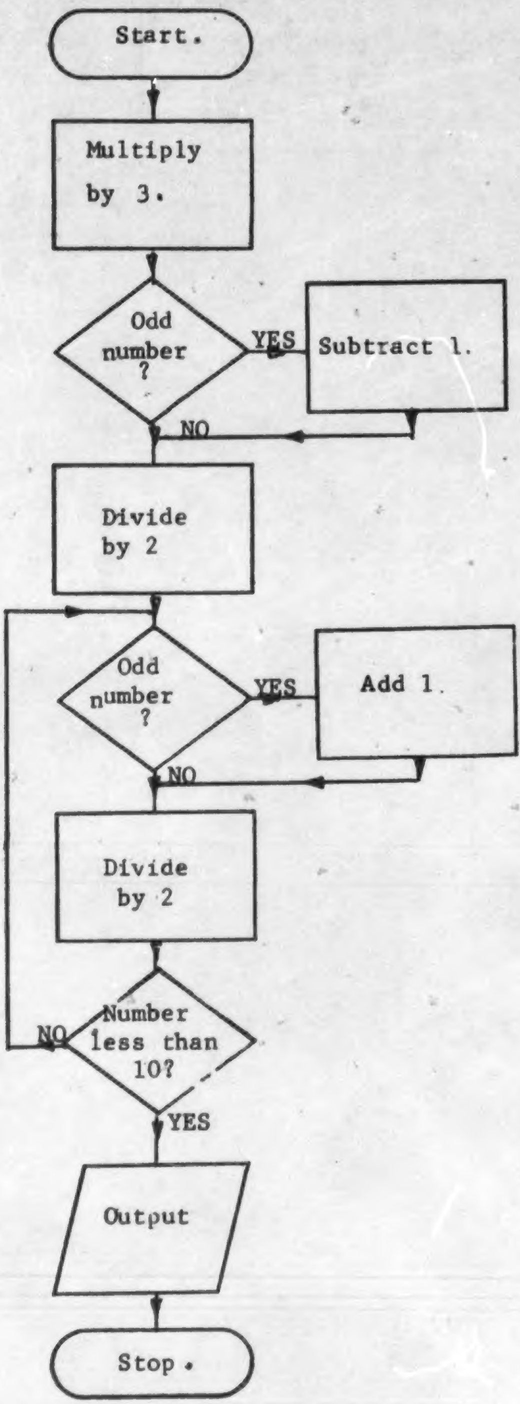
c) Given the following flowchart and Input, find the Output.


Start.
Input.
Add 1.
NO
Even number ?
YES
Number less than 10 ?
NO
Divide by 2.
YES
Number divisible by 3?
NO
YES
Output.
Stop

	Input	Output
1.	63	
2.	72	
3.	84	

PERFORMANCE OBJECTIVE XV-3 (continued)

d) Given the following flow chart and Input, find the Output.


Start.
Multiply by 3.
Odd number ?
YES
Subtract 1.
NO
Divide by 2
Odd number ?
YES
Add 1.
NO
Divide by 2
Number less than 10?
NO
YES
Output
Stop.

	Input	Output
1.	1	
2.	26	
3.	89	

353

PERFORMANCE OBJECTIVE XV-4

Draw a flowchart for a given sequence of instructions.

a) Construct a flowchart for the following set of instructions:

Start.

Read input.

Multiply the numbers.

Is the product divisible by 2?

Yes--Continue

No--Add 1 to product

Print output.

Stop.

b) Construct a flowchart for the following set of instructions:

How To Multiply Integers

Start.

Read Input.

Multiply absolute values.

Did the original numbers have the same sign?

Yes - Continue

No - Put "-" sign in front of answer.

Print Output.

Stop.

PERFORMANCE OBJECTIVE XV-4 (continued)

c) Construct a flowchart for the following set of instructions:

How To Find the Perimeter of a Polygon

```
Start.
Are all sides the same length?
    Yes - Count the number of sides
          Multiply the number of sides by the length of one side
          Go to Print
    No  - Continue
Add the lengths of all sides.
Print Output.
Stop.
```

d) Construct a flowchart for the following set of instructions:

How To Determine Whether a Number Is Divisible by 9

```
Start.
Read Input.
Add digits of number.
Is the sum divisible by 9?
    Yes - Continue
    No  - Number is not divisible by 9
          Go to Print
Number is divisible by 9.
Print Output.
Stop.
```

PERFORMANCE OBJECTIVE XV-5

Construct a flowchart which specifies the procedure for accomplishing a given task.

Construct an appropriate flowchart for each of the following tasks:

a) Getting a date

b) Adding a positive and negative integer

c) Buying a pair of slacks

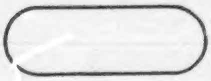
d) Finding the perimeter of a rectangle

Answers

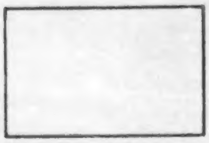
1. a) 1. Start
   2. Input
   3. Operation
   4. Decision
   5. Output
   6. Stop

   b) 1. C
   2. A
   3. D
   4. B

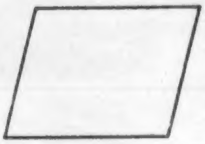
   c) 1.



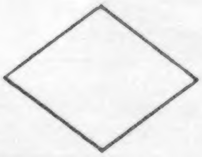
2.



3.

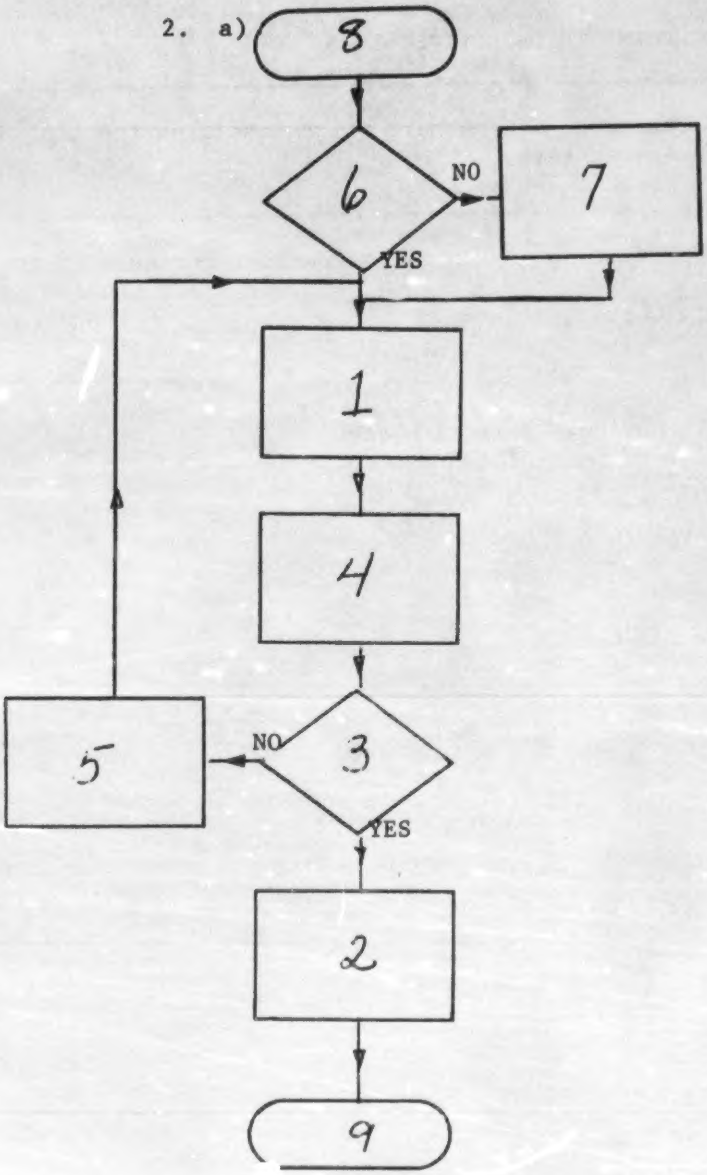


4.



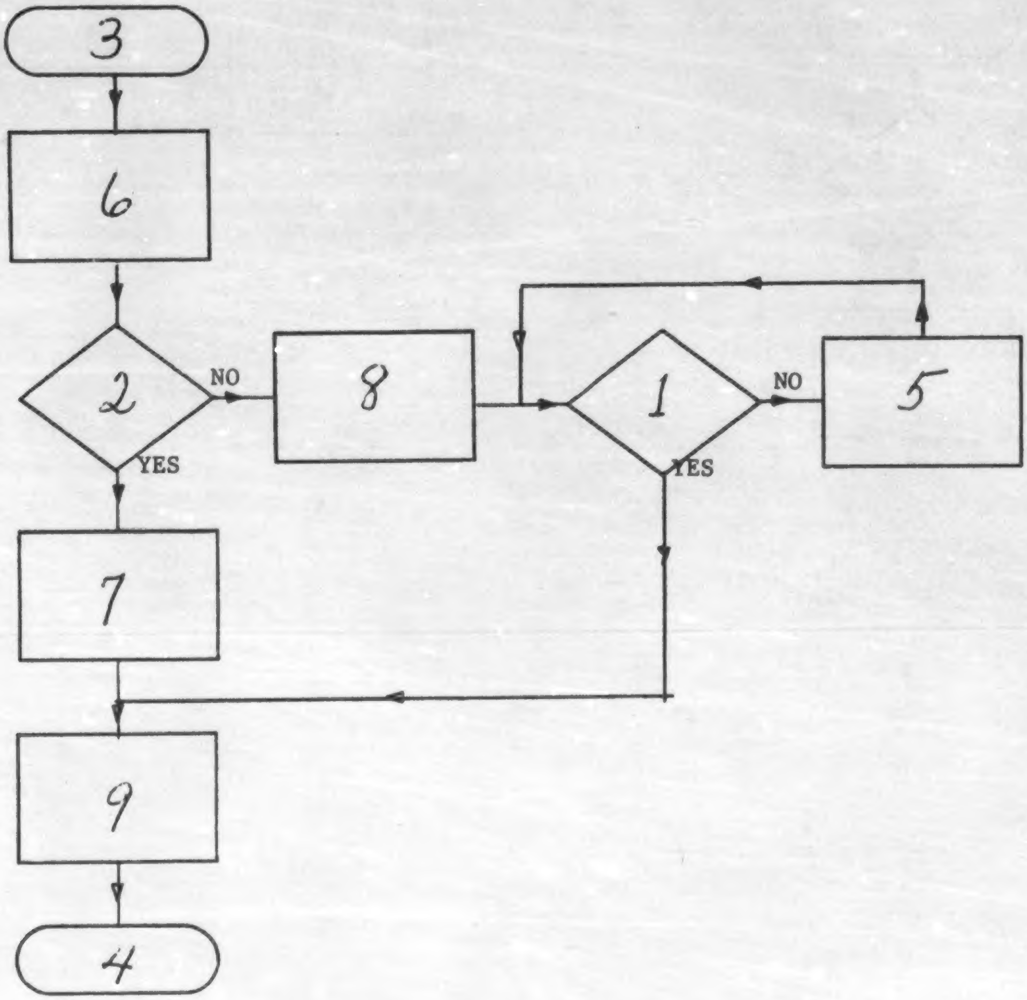
d) B

2. a)


8
6
NO
7
YES
1
4
NO
3
5
YES
2
9

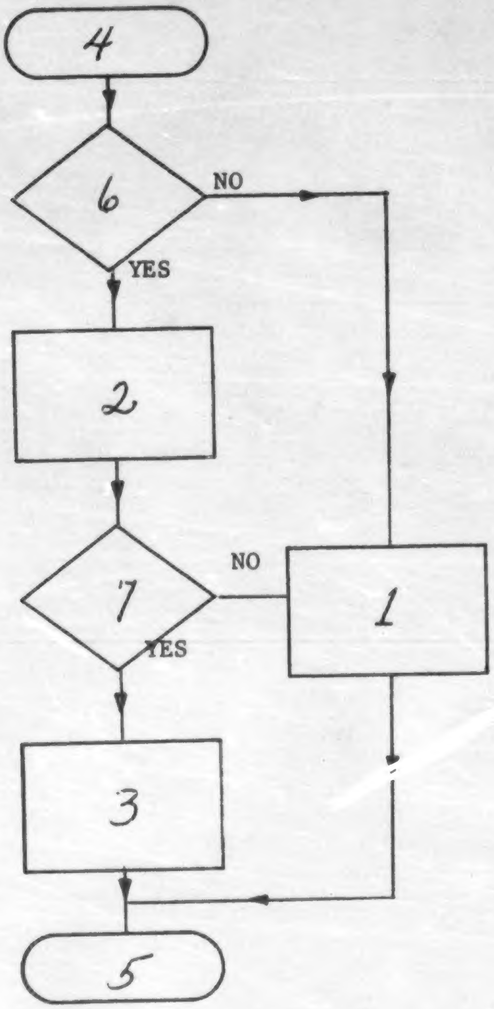
__Answers__ (continued)

2. b)

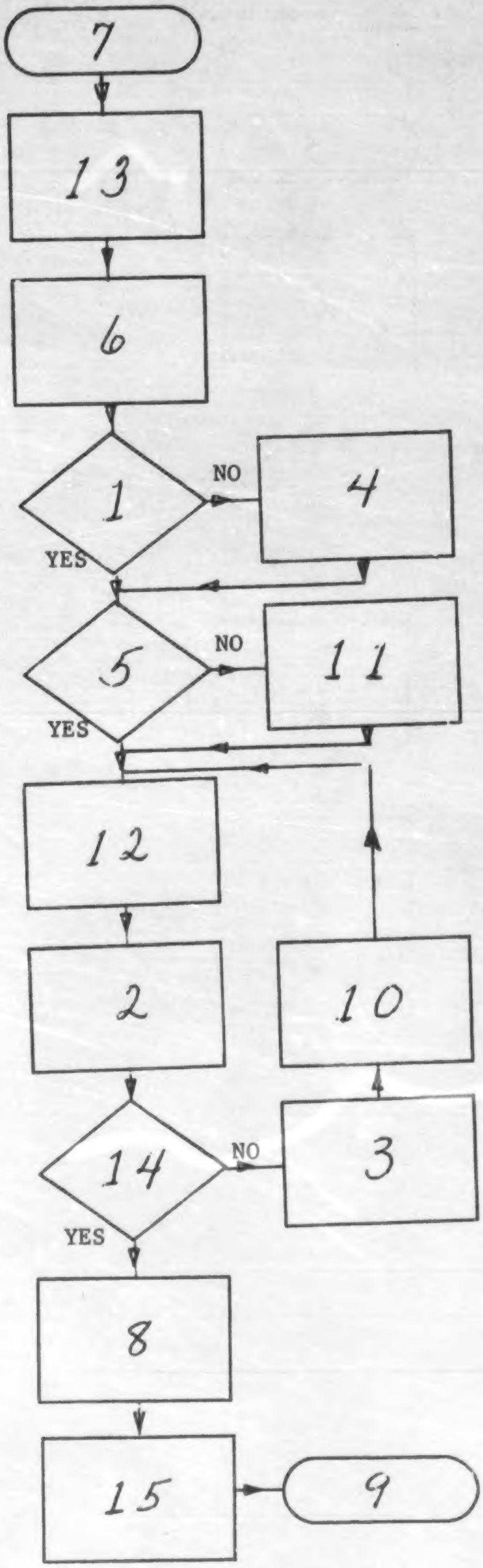

3
6
2
NO
YES
8
1
NO
YES
5
7
9
4

Answers (continued)

2. c)


4
6
NO
YES
2
7
NO
YES
1
3
5

d)


7
13
6
1
NO
YES
4
5
NO
YES
11
12
2
10
14
NO
YES
3
8
15
9

Answers (continued)

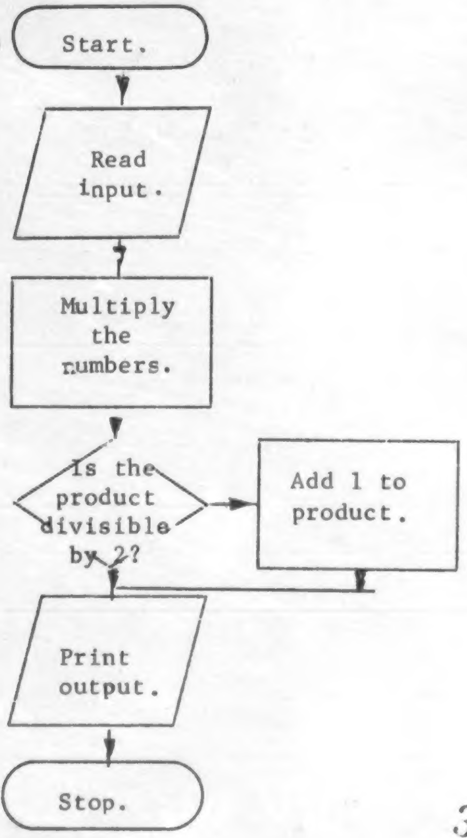
3. a) 1. 70
   2. 32
   3. 26

   b) 1. 10
   2. 20
   3. 30

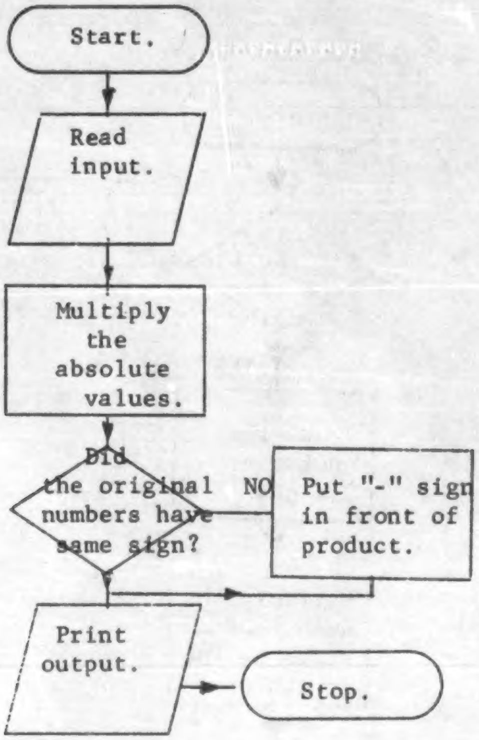
   c) 1. 9
   2. 6
   3. 6

   d) 1. 1
   2. 6
   3. 9

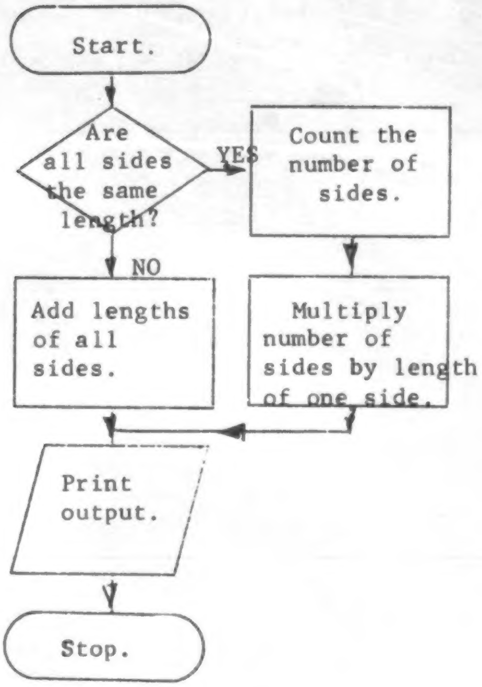
4. a)


Start.
Read input.
Multiply the numbers.
Is the product divisible by 2?
Add 1 to product.
Print output.
Stop.

4. b)

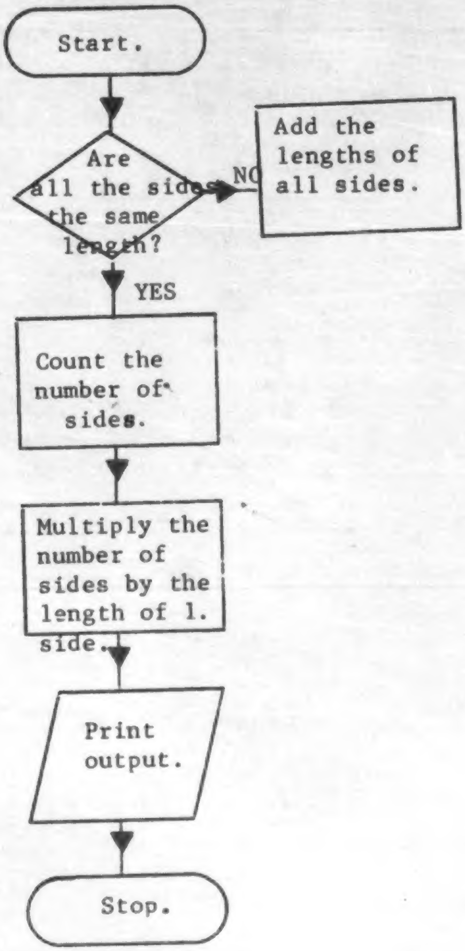

Start.
Read input.
Multiply the absolute values.
Did the original numbers have same sign?
NO
Put "-" sign in front of product.
Print output.
Stop.

c)

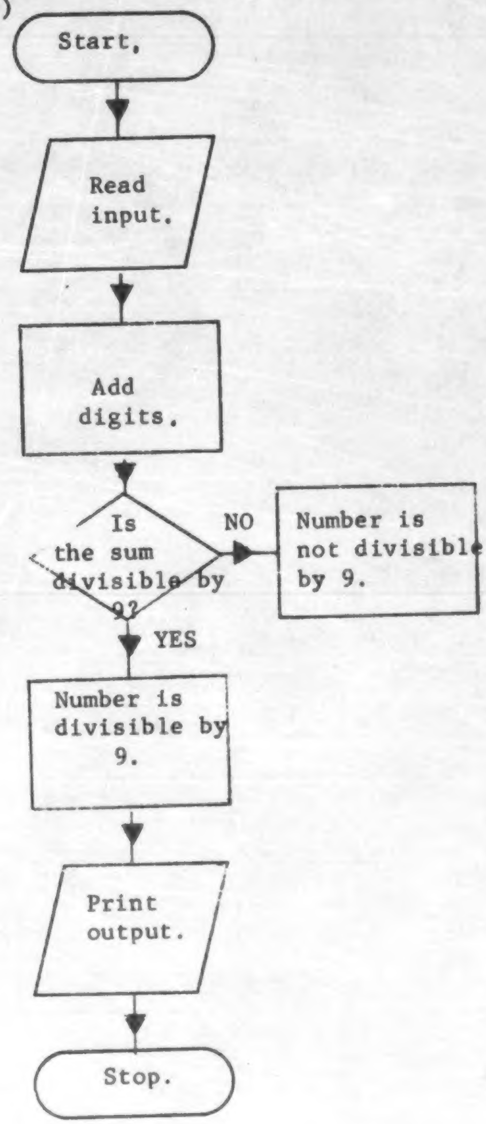

Start.
Are all sides the same length?
YES
Count the number of sides.
NO
Add lengths of all sides.
Multiply number of sides by length of one side.
Print output.
Stop.

Answers (continued)

4. c) Alternate Solution


Start.
Are all the sides the same length?
NO
Add the lengths of all sides.
YES
Count the number of sides.
Multiply the number of sides by the length of 1. side.
Print output.
Stop.

4. d)


Start,
Read input,
Add digits,
Is the sum divisible by 9?
NO
Number is not divisible by 9.
YES
Number is divisible by 9.
Print output.
Stop.

5. Answers will vary.